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A. H. Cook, M.A., Ph.D., F.R.A.S., F.G.S. (*Geophysical Secretary*), National Physical Laboratory, Teddington.

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Some Preliminary Measurements of Anisotropy of Magnetic Susceptibility of Rocks

R. W. Girdler

(Received 1960 November 21)

Summary

Measurements have been made on four sets of rocks, two of which showed anisotropy of magnetic susceptibility and two were isotropic. The maximum and intermediate principal susceptibilities lie in the cleavage plane for some Welsh slates and in the horizontal plane for some igneous rocks of the Skaergaard layered intrusion. In both cases the directions of natural remanent magnetization have been affected. The two sets of isotropic rocks are from the northern Pyrenees and Deccan of India respectively. Both may have been affected by stress environments, the nature of which did not cause measurable anisotropy. It is unlikely that the directions of natural remanent magnetization have been affected for these rocks.

1. Introduction

Apart from the classical work of Ising (1943) on the susceptibility anisotropy of the varve clays of Sweden, few detailed studies have been made of the anisotropy of magnetic susceptibility of rocks.

Graham (1954) found the maximum susceptibility lies in the bedding plane for some sediments but perpendicular to the bedding plane for some folded ferruginous sandstones. He also found some diabase dykes which showed 0·1 to 10 per cent anisotropy. Howell & others (1958) obtained similar results for sediments and found the plane of maximum susceptibility to lie in the foliation plane for some metamorphic rocks. Hargraves (1959) made a study of the susceptibility anisotropy and remanent magnetization of the Allard Lake haemo-ilmenite deposits of Quebec and found the plane of maximum susceptibility coincides with the preferred crystallographic grain orientation of titan-hematite and the remanent magnetization lies in or near this plane.

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At the Department of Geodesy and Geophysics, Cambridge, measurements of susceptibility anisotropy have been made on several different rock types using an a.c. bridge. With this apparatus differences of susceptibility of 2 per cent may be reliably measured. The experimental method and computational procedure have been described separately (Girdler 1961). For small fields, the anisotropy of susceptibility may be represented by an ellipsoid, the lengths of the principal axes being proportional to the maximum, intermediate and minimum principal susceptibilities. To help visualize the degree of anisotropy the results are presented by giving the ratios of the *maximum* : *intermediate* : *minimum* susceptibilities. Throughout, special reference is made to the effect of susceptibility anisotropy on the directions of natural remanent magnetization.

2. Welsh slates

The susceptibility anisotropy of two specimens of Welsh slates of Ordovician age was measured and the ratios of maximum/minimum and intermediate/minimum susceptibilities are given in Table 1. The directions of the three principal suscepti-

Table 1

Susceptibility anisotropy of Welsh slates

Specimen No.	Ratio Max. : Int. : Min.	X_{min} (e.m.u./g)
D–11–1	2·28 : 1·63 : 1	$2{\cdot}58\times10^{-4}$
D–14–1	1·80 : 1·44 : 1	$2{\cdot}69\times10^{-4}$

bilities are shown in relation to the rock cleavage on Schmidt Equal area projections in Figures 1(a) and 1(b). The maximum and intermediate susceptibilities are found to lie close to the cleavage plane and the minimum susceptibility is at right

FIG. 1.—Directions of principal susceptibilities and natural remanent magnetization for some Welsh slates.
The plane of the projection (Schmidt Equal Area) is horizontal; positive directions (downwards) are shown by dots and negative directions (upwards) are shown by open circles.

angles to the cleavage. The natural remanent magnetization was measured using an astatic magnetometer and was found to lie within 10° of the cleavage for both specimens. This work has been continued by Fuller who has obtained similar results for 50 specimens.

X-ray and Curie temperature studies (Fuller 1960) indicate that the main ferromagnetic constituent is pyrrhotite ($Fe_{1-x}S$) for slates with ratio of maximum/minimum susceptibility greater than 1·4. Microscopic examination showed the pyrrhotite to be in the form of platey grains lying parallel to the cleavage. As pyrrhotite has an easy plane of magnetization the observed anisotropy of the slates is most likely due to the alignment of pyrrhotite grains with their high susceptibility basal planes parallel to the cleavage.

3. Skaergaard rocks

The anisotropy of susceptibility has been measured for seven orientated samples from the Skaergaard intrusion of east Greenland, the structure and petrology of which have been described by Wager & Deer (1939). The rocks are ferro-gabbros, which contain pyroxenes and olivines abnormally rich in combined iron. The

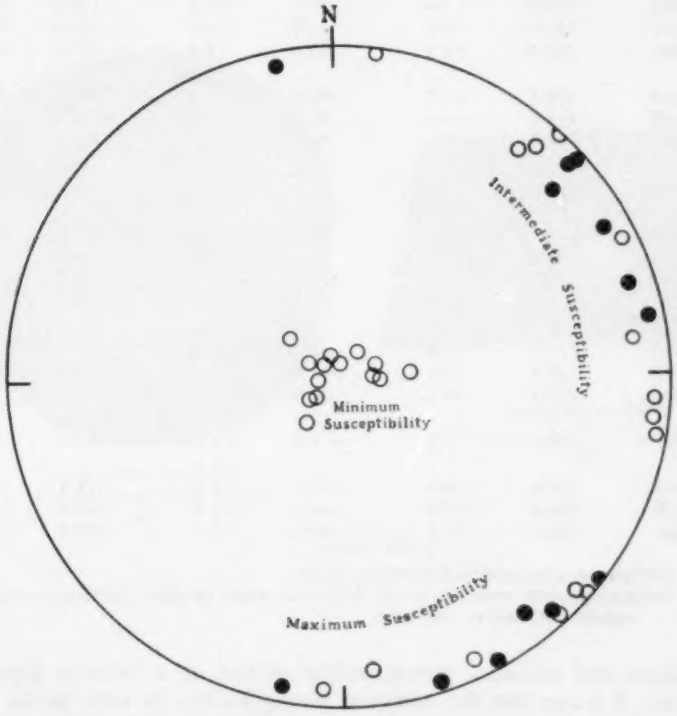


FIG. 2.—Directions of principal susceptibilities for Skaergaard Intrusive rocks. The plane of the projection (Schmidt Equal Area) is horizontal; positive directions (downwards) are shown by filled circles and negative directions (upwards) are shown by open circles.

Skaergaard intrusion is famous for its gravitational layering and the samples studied showed typical layered structure.

Fourteen cylinders were machined from samples provided by Professor Wager. For all specimens the minimum susceptibility is found to differ appreciably from the maximum and intermediate susceptibilities. The difference between maximum and intermediate susceptibilities is small. The mean differences are: maximum ~ minimum 8·6 per cent, intermediate ~ minimum 6·2 per cent, and maximum ~ intermediate 2·6 per cent. The smallest value of χ is $1\,450 \times 10^{-6}$ e.m.u./g.

The directions of the principal susceptibilities are given in Table 2, where the mean directions have been computed using the method described elsewhere by the author (Girdler 1961). Figure 2 shows the directions of the maximum,

Table 2

Directions of principal susceptibilities of Skaergaard Rock specimens

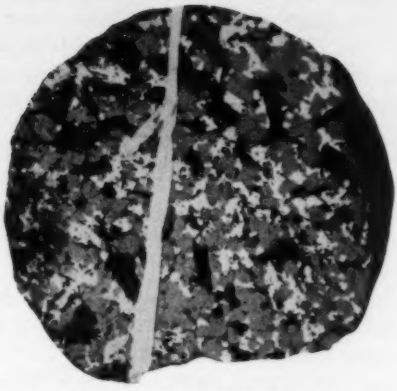
	Maximum		Intermediate		Minimum	
	θ	ϕ	θ	ϕ	θ	ϕ
3804A	138·6	+5·8	48·0	+6·1	91·4	−81·6
3804B	128·9	+1·0	39·1	−13·9	215·0	−76·1
3804C	131·9	−1·6	41·7	−9·4	231·4	−80·5
Mean	133·0	+2·1	43·2	−5·4	202·1	−84·2
3805A	175·8	−11·0	83·6	−11·2	309·5	−74·3
3805B	131·9	+1·9	61·6	+9·4	73·2	−80·4
Mean	169·8	−7·0	79·7	−0·8	342·9	−82·3
3806	155·8	−6·2	65·2	−5·4	294·2	−81·9
3807A	142·7	+10·5	49·9	+14·4	87·7	−72·1
3807B	162·9	+3·3	72·4	+8·6	93·9	−80·8
3807C	80·2	+5·8	349·7	+5·3	37·5	−82·2
Mean	155·8	+2·1	65·4	+10·3	77·2	−79·4
3808A	133·2	−4·4	43·1	−0·6	305·7	−85·6
3808B	138·1	−2·3	48·2	+2·4	3·5	−86·7
Mean	136·5	−3·3	46·6	−0·8	329·8	−86·6
3809	98·1	−5·3	7·9	−1·8	259·3	−84·4
3811A	191·4	−4·4	101·1	−3·4	334·1	−84·5
3811B	183·9	+5·8	94·5	−5·6	228·2	−82·0
Mean	186·6	+1·5	96·9	−4·2	257·8	−85·5

θ = Declination, measured clockwise from north.
ϕ = Inclination, with reference to the horizontal plane (positive inclination—downwards, negative inclination—upwards).

intermediate and minimum susceptibilities plotted on a Schmidt Equal Area projection. It is seen that the minimum susceptibilities are close to the vertical and the maximum and intermediate susceptibilities are all close to the horizontal plane but are widely scattered within it. Although the maximum and intermediate susceptibilities show considerable scatter, they lie within 65° of arc, the maximum in the south-east quadrant and the intermediate in the north-east quadrant.

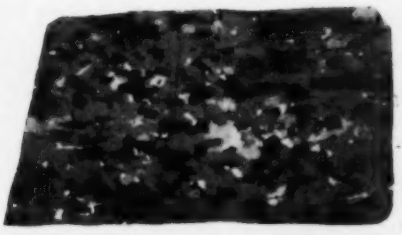
PLATE I

Specimen no. 3805 B
Horizontal polished section
(a)



Specimen no. 3807 B
Horizontal polished section
(b)

Specimen no. 3809
Horizontal polished section
(c)



Specimen no. 3809
Vertical polished section
(d)

1·0 cm.

[facing p. 200

The scatter of the directions of principal susceptibilities is related to the differences between their magnitudes. Since the minimum susceptibility is considerably different from both the maximum and intermediate susceptibilities, the directions of minimum susceptibility are closely grouped; but since the magnitudes of the maximum and intermediate susceptibilities are nearly equal, their directions are widely scattered in the horizontal plane.

The susceptibility anisotropy is consistent with the petrology and fabric of the rocks. Vincent & Phillips (1954) have studied the opaque minerals and Chevallier & others (1954) show that cubic titano-magnetite is the main ferromagnetic constituent. As titano-magnetite is magnetically isotropic, the observed anisotropy should be due to the shape and distribution of these grains. Plate I shows photographs of four polished sections. (a), (b) and (c) are horizontal sections and (d) is a vertical section. As titano-magnetite has high reflectivity, the grains appear white and their shape can be easily studied. The horizontal sections reveal that the titano-magnetites are approximately equidimensional in the horizontal plane but the vertical section shows that the grains are flattened and may be distributed in layers. Measurements have been made of the grain shape (Uyeda & others 1961) and it was found that the observed anisotropy is in excellent agreement with that calculated from the shape of the grains. It is concluded that the susceptibility anisotropy of the Skaergaard rocks is due to the preferred alignment of slightly oblate titano-magnetite grains in the horizontal plane.

The natural remanent magnetization was measured for the fourteen specimens and all were found to be reversely magnetized. The mean directions for each sample are shown by open circles in Figure 3.

Uyeda (see Uyeda & others 1961), carried out a series of heating experiments on a set of magnetite disks and on one of the Skaergaard specimens. The specimens were heated in a nitrogen furnace to above their Curie temperatures and cooled in the present geomagnetic field. For the magnetite disks, the thermal remanent magnetization deviates from the direction of the external field towards the plane of maximum susceptibility, the deviation increasing as the shape factor increases. A similar effect was observed for the Skaergaard specimen, the deviation from the ambient field being approximately 5°. Thus it was found that the directions of n.r.m. do not represent the direction of the Earth's magnetic field at the time of cooling of the rock.

The deviation can also be studied by a graphical consideration of the directions of natural remanent magnetization and principal susceptibilities of all the samples. It has been shown experimentally that the direction of thermal remanent magnetization migrates away from the direction of the ambient field towards the plane of maximum susceptibility. Therefore, it may be assumed that a great circle passing through the directions of remanent magnetization and minimum susceptibility should pass through the ambient field direction. As the degree of preferred orientation is different for each sample the effect will differ and a set of great circles will be obtained which should ideally intersect at a common point which is the direction of the ambient field at the time of cooling of the rock. This has been done in Figure 3 where the directions of n.r.m. and minimum susceptibility for each sample have been joined by great circles. The mean directions and 95 per cent circles of confidence (Fisher 1953) have been computed for the directions of n.r.m., minimum susceptibility and intersections of the great circles and these are illustrated in Figure 4. It is seen that the mean direction of natural remanent magnetization differs from the direction of the ambient field by

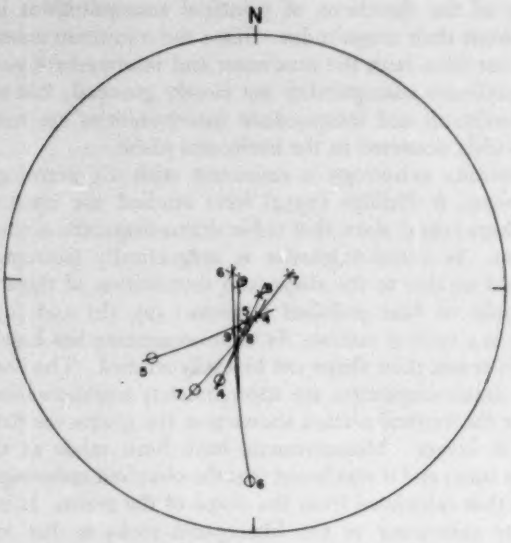


FIG. 3.—Great circles joining mean directions of natural remanent magnetization and mean directions of minimum susceptibility for each sample.

The plane of the projection (Schmidt Equal Area) is horizontal; directions of n.r.m. (negative) are shown by open circles and directions of minimum susceptibility (upwards) are shown by crosses.

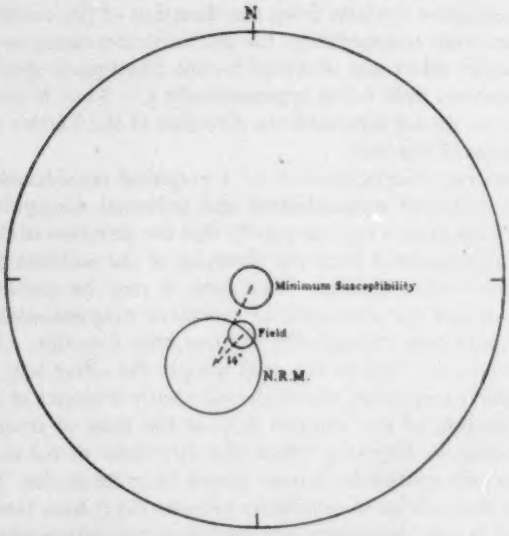


FIG. 4.—Relationship between natural remanent magnetization, minimum susceptibility and ambient field for Skaergaard rocks.

approximately 14°. It is also noted that although the difference between minimum and maximum susceptibilities is sufficient to cause a deviation of the direction of n.r.m. from that of the ambient field, it seems from Figure 4 that the difference between maximum and intermediate susceptibilities is too small to cause any appreciable effect.

4. Lower Jurassic volcanic rocks from the Northern Pyrenees

The susceptibility anisotropy has been measured for 25 samples of some igneous rocks from the northern foothills of the Pyrenees. These were chosen as part of the preliminary study to see if igneous rocks which have been subjected to orogenesis show preferred directions of susceptibility anisotropy. If any preferred directions exist, it seemed possible that these would be related to the Alpine fold directions of the Pyrenees.

Samples were collected from two areas, the first is located west of Foix (latitude 42° 57′ N, longitude 1° 35′ E) and the second is south-east of Bayonne (latitude 43° 30′ N, longitude 1° 28′ W). The relevant geology of the region is summarized in Girdler (1958) where the directions of natural remanent magnetization for rocks from the first locality are given. The age of the rocks is Lower Hettangian, i.e. equivalent to the Upper Lias of the English Jurassic System.

The rocks collected from four sites west of Foix are volcanic tuffs containing fragments of basalts, limestones and marls. The mean anisotropy for 11 rock cylinders is:

$$\text{maximum} : \text{intermediate} : \text{minimum} = 1{\cdot}024 : 1{\cdot}009 : 1$$

and the minimum susceptibility is $1\,550 \times 10^{-6}$ e.m.u./g. The directions of maximum susceptibility show no significant preferred orientation either before and after corrections have been applied for geological strike and dip. This is to be expected as the rock specimens are almost isotropic and the directions of their principal susceptibilities are therefore poorly determined.

At the second locality (south-east of Bayonne) samples were collected from a basaltic lava flow at two sites. Of the 18 samples, 13 were found to be reversely magnetized. The mean anisotropy of susceptibility for 15 rock cylinders is:

$$\text{maximum} : \text{intermediate} : \text{minimum} = 1{\cdot}065 : 1{\cdot}034 : 1$$

and the minimum susceptibility $\chi = 520 \times 10^{-6}$ e.m.u./g. Again, the directions of principal susceptibilities show no significant preferred orientation agreeing with the very small degree of anisotropy.

It is concluded that the pressures and temperatures associated with the Alpine folding did not cause a significant shape or crystallographic alignment of the ferromagnetic minerals.

5. Deccan Trap Rocks of India

Clegg and his co-workers in London have made a magnetic study of the Deccan Trap rocks of India. The rocks are mainly basaltic lavas and range in age from Eocene to possibly late Jurassic.

Studies of the natural remanent magnetization of these rocks have been described in Clegg & others (1956), Deutsch & others (1958) and Clegg & others (1958). Their results are summarized in stratigraphical order in Table 3.

Table 3

Mean directions of natural remanent magnetization for Deccan Trap Rocks

Site	Locality	θ	ϕ
I	Upper Pavagadh rhyolite tuffs	355	+10
II–IV	Upper Nipani, Upper Amba and Lower Pavagadh	341	−24
V–VIII	Lower Nipani, Lower Amba, also Linga and Khandala	156	+56
IX	Rajmahal Traps (? Jurassic)	328	−64

θ = Declination measured clockwise from north.
ϕ = Inclination, with reference to the horizontal plane (positive inclination—downwards, negative inclination—upwards).

It is seen that the directions of natural remanent magnetization show a systematic decrease of inclination (ϕ) with altitude. Clegg & others (1958) give three possible causes for this. First, it may have occurred as a result of some physical phenomenon associated with different depths of burial such as magneto-striction (Graham 1957). Secondly, it may be due to partial magnetic instability which would tend to pull the directions towards the present field. This would tend to increase the inclinations of the reversely magnetized rocks (V–VIII) and decrease those of the normally polarized ones (I–IV and IX). Thirdly it may represent a continuous northward movement of India relative to the Equator during the time of eruption of the Deccan Traps.

For various reasons, Clegg & others conclude that the combined results for the Rajmahal and Deccan Traps conform best to the third postulate of a continuous northward movement of the Indian landmass from Jurassic to Cretaceous times.

To investigate further the first possibility, the susceptibility anisotropy has been measured for 26 specimens which represent a sampling throughout the whole succession of lavas. It was hoped to reveal any systematic variation of susceptibility anisotropy corresponding to the variation in magnetic inclination and if possible to relate this to the depth of burial.

The results are given in Table 4. The value of χ ranges from 285 to $3\,530 \times 10^{-6}$ e.m.u./g. The largest difference between maximum and minimum susceptibilities is found to be 3·6 per cent and the mean percentage difference between maximum and minimum susceptibilities is 1·6 per cent. Inspection of the ratios of the principal susceptibilities shows that the susceptibility is almost isotropic and the directions of the principal axes are therefore almost random.

It is concluded that the stresses associated with the depth of burial of the lavas were insufficient to cause shape or crystalline magnetic anisotropy and it is unlikely that the directions of natural remanent magnetization have been significantly affected by this cause.

6. Conclusions

Measurements for susceptibility anisotropy have been made for four sets of rocks. Two of these showed anisotropy whilst the other two were isotropic within the limits of the measuring apparatus. Of the two sets of anisotropic rocks, it is established that the anisotropy is due to different causes; to crystalline effect and shape effect respectively.

Table 4

Susceptibility anisotropy for a selection of Deccan Trap specimens

Specimen Number	Anisotropy			Direction of Maximum k	
	Max. :	Int. :	Min.	θ	ϕ
A24	102·1	101·8	100	217	−35
K14a	102·1	100·2	100	134	−66
K29a	102·8	101·6	100	70	−78
K36aI	103·6	103·4	100	290	−41
K52a	103·5	102·2	100	347	−44
K56a	101·2	100·7	100	284	+37
K64aI	101·5	101·0	100	28	+28
K80aI	102·0	100·9	100	184	−55
K81a	101·4	101·0	100	131	−3
K82a	100·9	100·8	100	357	−37
K87a	100·8	100·4	100	159	−33
K95aI	100·7	100·3	100	152	−53
K92a	101·2	100·9	100	299	−50
K98aI	101·1	100·6	100	216	+8
K107aI	100·6	100·4	100	53	−16
K114a	101·0	100·8	100	32	−11
K116a	100·9	100·4	100	357	−66
K119a	102·3	101·1	100	156	+42
L49BI	101·2	100·3	100	32	+8
L58a	100·6	100·3	100	30	−29
L59a	101·9	101·6	100	42	+34
L64a	101·0	100·4	100	49	+34
L67a	101·0	100·5	100	70	−81
N34I	101·8	101·0	100	232	−31
R231	102·3	100·4	100	71	+4
R27	102·4	100·8	100	250	−13

θ = Declination of maximum susceptibility, measured clockwise from north.
ϕ = Inclination of maximum susceptibility with reference to the horizontal plane.

For the Welsh slates the maximum susceptibility, intermediate susceptibility and natural remanent magnetization all lie close to the cleavage plane. The susceptibility anisotropy is due to the preferred alignment of pyrrhotite grains which possess an easy plane of magnetization.

For the Skaergaard rocks, the maximum and intermediate susceptibilities lie close to the horizontal plane and the anisotropy is found to be due to the shape and distribution of isotropic titano-magnetite grains. The natural remanent magnetization of these rocks deviates from the ambient field but it is possible to determine accurately the direction of the past field from a knowledge of the susceptibility anisotropy and the directions of natural remanent magnetization. It is noted that anisotropy may be a factor contributing to the scatter in directions of natural remanent magnetization for some igneous rocks (Figure 3).

For the Pyrenees and Indian rocks, it is concluded that conditions of pressure and temperature during their history have been insufficient to cause susceptibility anisotropy and the natural remanent magnetization has been unaffected by this cause. This does not exclude the possibility that the remanent magnetization may have been affected by some other mechanism.

In general, the fabric of igneous rocks will have been attained at temperatures higher than the Curie temperature of the ferromagnetic minerals which may be present. If susceptibility anisotropy exists, the natural remanent magnetization will contain a component of the ambient field and it is possible to determine the direction of the Earth's past field from a knowledge of the susceptibility anisotropy and natural remanent magnetization. Conversely, if a rock has been subject to temperatures higher than the Curie temperature of the minerals present during deformation, the natural remanent magnetization will contain a component of the Earth's field during the time of deformation. It is likely that similar considerations apply to chemical remanent magnetization.

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Department of Geology,
Science Laboratories,
South Road,
Durham City:
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The Granitic Layer

M. H. P. Bott

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Summary

The negative gravity anomalies observed over granite masses show that the upper part of "the granitic layer" is consistently denser than granite. Evidence is presented to suggest that the lower part may be closer to granite in composition, and thus both less dense and more siliceous than the overlying part. Two seismological tests for the suggested subdivision are proposed.

1. Introduction

One of the outstanding problems of crustal* geophysics concerns the chemical constitution and layering of the crust in continental regions. Three lines of evidence have been especially important in discussing this problem. The first, the seismological approach, has met with the greatest success, for in recent years knowledge of the thickness and variation of thickness of the crust has become well established from studies on waves generated by earthquakes and artificial explosions. There is also considerable seismological evidence to support the existence of the Conrad discontinuity (i.e. the boundary between the upper and intermediate layers); and the continental crust is found to act as a waveguide for certain short-period elastic waves. Secondly, the evidence from gravity measurements has shown that the crust of the Earth as a whole is approximately in isostatic equilibrium, with certain well-known exceptions. Thirdly, important evidence comes from the study of igneous rocks which are normally thought to come from the crust or the upper mantle.

Each of these approaches has its characteristic limitations. Interpretation of seismological information can be ambiguous for relatively thin layers with small velocity contrasts and may be hampered further by lateral inhomogeneities within the crust. Petrological studies are limited by the possibility that even the most abundant igneous rocks, such as basalt and granite, may be formed by differentiation or selective melting from a variety of parent materials.

In this paper the earlier concept of "the granitic layer" is critically examined in the light of the large negative Bouguer anomalies observed over granite masses. It is shown that the upper part of this layer is considerably denser than either granite or granodiorite. On the other hand, evidence is presented to suggest that the lower part may be more close to granitic composition.

* "Crust" is taken throughout the paper to indicate the region overlying the Mohorovičić discontinuity.

2. Definition of terms

The geological terms used in the paper are defined in this section. Igneous rocks are divided into four groups depending on their silica percentage:

(i) *acid rocks*—more than 66 per cent SiO_2
(ii) *intermediate rocks*—between 52 per cent and 66 per cent SiO_2
(iii) *basic rocks*—between 45 per cent and 52 per cent SiO_2
(iv) *ultrabasic rocks*—less than 45 per cent SiO_2.

In addition igneous rocks are grouped into families depending on their mineral content. The families concerning us are (Turner & Verhoogen 1960):

(i) *granite-granodiorite family*—coarse-grained acid rocks consisting essentially of quartz and feldspar, with hornblende and/or mica as typical dark minerals. Granite (*sensu strictu*) has a predominance of potash feldspar over sodic plagioclase, while plagioclase is predominant in granodiorite.
(ii) *diorite family*—coarse-grained igneous rocks of intermediate composition consisting essentially of plagioclase feldspar (variety andesine) and hornblende.
(iii) *basalt family*—fine-grained basic igneous rocks containing plagioclase, augite and frequently olivine. The olivine-free types are known as "tholeiites".

In dealing with the crustal layers, chemical composition is of greater importance than the mineral content or grain size of the component rocks. However, in line with current geophysical usage the terms "granitic", "dioritic" and "basaltic" are used to describe rocks with physical properties of the acid, intermediate and basic groups respectively, irrespective of grain size or mode of origin. Thus the term "granitic rocks" is used to refer to large bodies of rocks with more than 66 per cent SiO_2.

To avoid a possible confusion between intermediate position and intermediate igneous rock group, the "intermediate layer" (or "basaltic layer") of seismology will hereafter be called the **"lower crustal layer"**. Similarly the "upper layer" (or "granitic layer") will be referred to as the **"upper crustal layer"**.

3. The seismological evidence

Some of the significant recent conclusions from seismology concerning the thickness and layering of the continental crust may be summarized as below:

(1) Thickness: seismic refraction measurements based on surface explosions (Gutenberg 1959, pp. 32–35), supported by surface wave studies (e.g. Ewing & Press 1959), suggest a normal crustal thickness in continental regions of between 30 and 40 km, with a thicker crust (50 to 60 km) underlying mountain ranges.

(2) Layering: it seems that there is general agreement on an overall increase in compressional velocity with depth in the continental crust. Some recent studies using explosions or earth tremors have also supported the concept of an upper crustal layer with compressional velocity of about 6·0 km/s overlying a lower crustal layer (the "intermediate layer") with velocity about 6·7 km/s, separated by the Conrad discontinuity. Particularly convincing evidence for the "intermediate layer" is given by Hales & Sacks (1959) for Eastern Transvaal, where earth tremors give strong and clearly defined second arrivals with a velocity of about

6·7 km/s. Lack of detection of the Conrad discontinuity by other workers (e.g. Tatel & Tuve 1955) may be due to a gradual change of composition between the layers or alternatively due to absence of any first arrivals from this discontinuity.

On the basis of a relatively low compressional velocity and rapid attenuation of the direct wave from near-earthquakes at depths of 15 km and more, Gutenberg (1951) suggested a decrease in velocity with depth in the upper crustal layer. Some possible support for this interpretation comes from the channel waves named *Lg* (Press & Ewing 1952), and especially *Lg*2 (Båth 1958), although these channel waves can be otherwise interpreted.

4. The evidence from gravity anomalies over granites

Evidence of a different type comes from a study of the low Bouguer anomalies characteristically observed over granite masses. The anomalies usually drop between 15 and 70 mgal over granite batholiths and it is well established (Bean 1953, Bott 1953, 1954, 1956, Innes 1960 and others) that the anomalies are essentially caused by the relatively low density of the granite in relation to the surrounding rocks. An example of a gravity profile across an exposed granite (from southern England) together with one possible interpretation is shown in Figure 1.

The negative anomalies have significance to the problems of origin and mode of emplacement of granites (Bott 1956) but these aspects are not discussed here

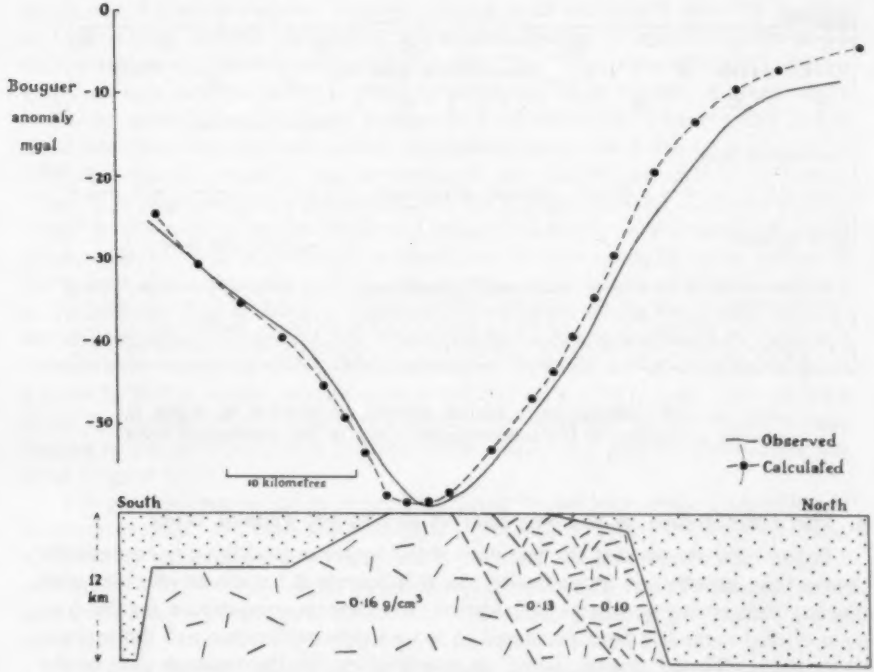

FIG. 1.—An observed gravity profile across the Bodminmoor granite, south-west England, compared with the anomalies calculated for the two-dimensional model shown (using electronic computing methods).

except where relevant to crustal structure. The two main facts of direct concern are:

(i) The near-surface rocks of the upper crustal layer are consistently denser than either massive granite or granodiorite, by about 0·10 g/cm³ on average. The density difference is greater for true granite than for granodiorite.

(ii) For British granites (Bott, Day & Masson-Smith 1957; Bott & Masson-Smith 1960) the density contrast from measured samples at the surface needs to extend to depths of 10 km or greater to account for the amplitude of the anomalies. This approaches a third of the normal thickness of the continental crust and more than half the thickness of the upper crustal layer. Variations in density contrast at depth are unlikely to influence the order of magnitude of the estimate. The writer has not made detailed studies of granites elsewhere but the general similarity of the amplitude of anomalies over granites suggests that the conclusion here is generally valid.

These two conclusions are illustrated diagrammatically in relation to the seismological concept of the continental crust in Figure 2. The implications will be discussed in the following section.

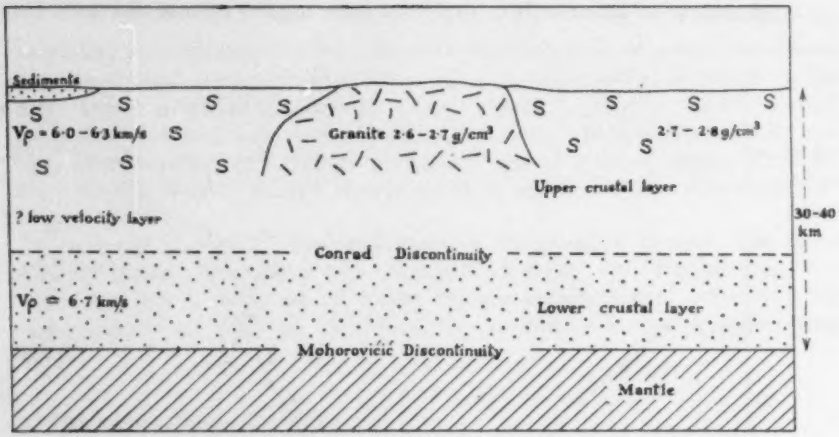

FIG. 2.—The relation of a typical granite, as revealed in depth by gravity anomalies, to the seismological model of the continental crust.

5. The constitution of the top part of the upper crustal layer

It has been shown that the top part of the upper crustal layer is consistently denser than granite and granodiorite. In this respect it has closer affinities with diorite. Supporting geochemical evidence for a dioritic composition for the topmost crustal rocks has been presented in the writer's earlier study of the probem (Bott 1953). Thus "granitic layer" is a misnomer for the topmost part of the sub-sedimentary continental crust and the terms "meta-sedimentary zone" (adapted from Bott 1953) or "dioritic zone of the upper crustal layer" might be less misleading.

The conclusion that the topmost continental crust is not granitic is readily understood if the geological processes forming these rocks are considered. These rocks, into which granites are emplaced, are either metamorphic rocks derived from earlier sedimentary and igneous rocks, or fully compacted sediments. All sedimentary rocks have ultimately been derived from igneous sources, of which the most important are granites and basalts, or from the primitive crust. Thus sedimentary rocks would be expected to show a bulk composition intermediate between basalt and granite, a characteristic which will naturally extend to metamorphic rocks. Hence we can expect the average composition of the topmost crustal rocks to be intermediate between granitic and basaltic, i.e. dioritic.

6. The constitution of the bottom part of the upper crustal layer

The density contrast between granites and their surrounding rocks appears to vanish at a depth shallower than the Conrad discontinuity. The problem now arises as to the composition of this lower portion of the upper crustal layer. One suggestion (i) is that granites merge at a depth of about 10 km into a zone of granitic composition forming the lower part of the upper crustal layer. An alternative suggestion (ii) is that granites are floored by rocks denser than themselves and similar in composition to the surrounding upper part of the layer. The first suggestion is favoured here.

The principal reason for preferring (i) comes from considering the possible origin of the granitic magma. Granite magma could be formed either by melting of a subsurface granitic zone or by differential melting of a dioritic zone or by differentiation of a basaltic magma of tholeiitic type. The main difficulty involved in a dioritic or basaltic source is that a considerably larger volume of source rocks would be needed than for direct melting of a granitic zone (Figure 3(a)). It has been calculated that the total volume of granite magma which can be produced by differentiation of a basaltic magma represents only about 5 per cent of the parent. Thus to produce sufficient granite magma for a large batholith of 10 km vertical extent would need a basaltic magma chamber within the upper mantle of about 40 or 50 km in vertical thickness of about four or five times the areal extent of the granite and frequently without any significant indication of basaltic activity at the surface. The problem is less acute for a dioritic source for granite magma, but still significant (Figure 3(b)). Since diorite and meta-sedimentary rocks are considerably poorer in alkalis than granite, to produce sufficient magma for a granite batholith would require partial melting of a dioritic zone over at least twice the volume of the resulting granite. The difficulty of producing granite magma in adequate volume is obviated if the source is a granitic zone within the crust (Figure 3(c)).

The granitic zone would be expected to form the top layer of the pre-geological continental crust, and thus the boundary separating it from the overlying meta-sedimentary zone represents the primitive surface of the crust. This granitic zone could contribute to early Precambrian quartz rich sediments by direct erosion, and this simple process seems to provide a satisfactory answer to the problem of the source of sedimentary quartz in Archean rocks.

The idea that the bottom part of the upper crustal layer is less dense than the overlying portion is fully consistent with the idea of a low-velocity channel within the crust. Since, however, the low-velocity channel has not been definitely established, this line of evidence can only be taken as tentative.

7. The origin of granite magma in relation to crustal structure

It needs to be established that temperatures sufficiently high to cause melting can be attained within the postulated granitic zone. Thus the steady state temperature distribution with depth (Figure 4) has been calculated for a three-layered crust consisting of an upper metasedimentary zone (with thermal conductivity of 0·0059 cal/cm s deg C), a granitic zone in the middle (conductivity 0·005) and

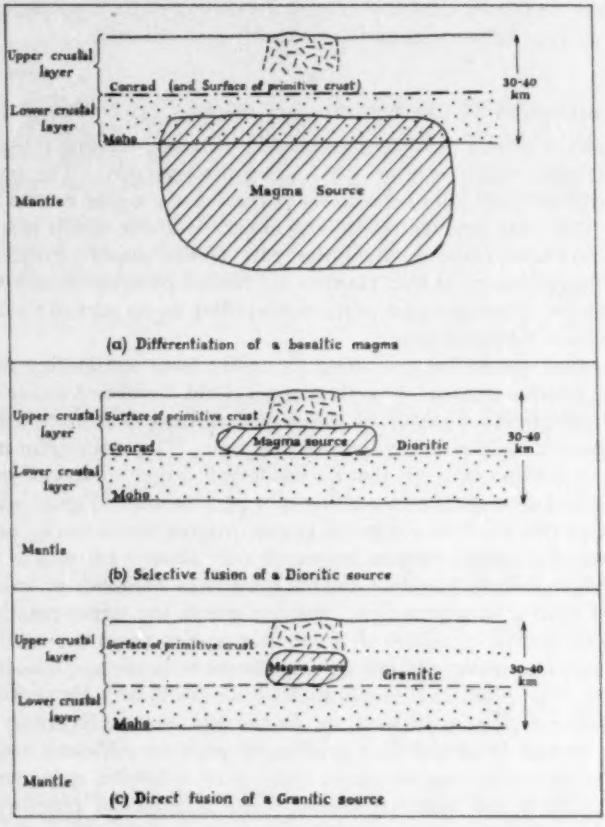

FIG. 3.—Hypotheses for the origin of granite magma in relation to possible models of the continental crust, showing the volume of magma chamber needed.

a lower crustal layer of basaltic composition (conductivity 0·005), each representing a third of the total crustal thickness. The rates of radioactive heat production given in Table 1 (based on Birch 1955) have been adopted and the heat flow into the base of the crust is taken as $0{\cdot}3 \times 10^{-6}$ cal/cm^{2}s. The temperature distribution has been calculated for three total crustal thicknesses; it is seen that the temperature at the base of the granitic zone is approximately proportional to the square of the total crustal thickness.

Table 1

Table showing the suggested subdivisions of the continental crust and their principle geological and physical characteristics

Layer	Boundary	Normal thickness km	Type of rock	Chemical composition	Density g/cm³	Velocity km/s	Rate of heat production cal/cm³ s
Sedimentary Layer		0–5	Sedimentary rocks, not fully compacted; possibly interbedded volcanic rocks	Intermediate	< 2·65	Variable	?
	Usually nonconformity						
Upper Crustal Layer — Dioritic zone		≈ 10	Fully compacted sedimentary rocks; interbedded volcanic rocks and intrusive granites; metamorphic varieties of these.	Intermediate	2·70–2·80	6·0–6·3	$\sim 3{\cdot}5 \times 10^{-13}$
	Surface of primitive crust						
Upper Crustal Layer — Granitic zone		5–15	Granitic or granodiorite rocks formed during pre-geological history of the Earth	Acidic	2·60–2·70	< 6·0	$\sim 6 \times 10^{-13}$
	Conrad discontinuity						
Lower Crustal Layer		10–15	Basaltic and possibly some andesite rocks formed during pre-geological history of the Earth	Basic	2·90–3·00	6·7 +	$\sim 1 \times 10^{-13}$
	Moho discontinuity						

The temperatures at which granite can melt at various depths are also shown on Figure 4 (taken from Tuttle & Bowen 1958). Apparently the granitic zone within a continental crust of normal thickness is nowhere at a sufficiently high temperature to melt. On the other hand a granitic zone with a high enough water content could melt completely when the normal thickness has been doubled, as in recently formed mountain belts. It is also possible that granite magmas could be formed under more normal conditions in early Precambrian times when the rate of heat production was greater, and perhaps more recently in regions where the rate of heat flow is anomalously high. These predictions are in agreement with the observed distribution of granite batholiths in space and time.

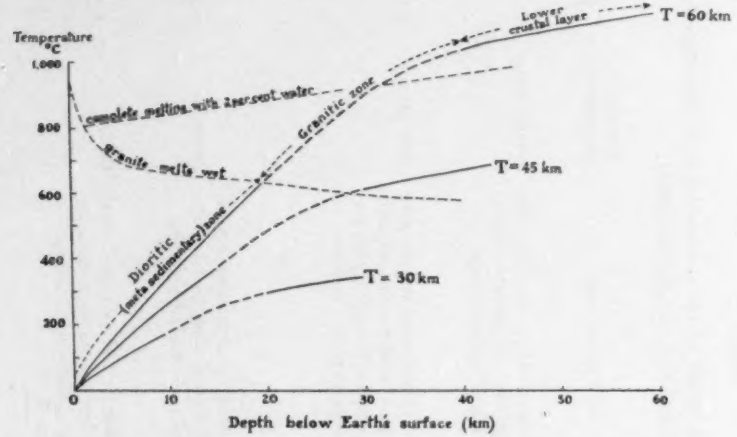

FIG. 4.—The distribution of temperature with depth in the crust in relation to the total crustal thickness and the melting temperature of granite (after Tuttle & Bowen). The dashed portion of each curve represents the granitic zone of the upper crustal layer. The basic assumptions of thermal conductivity etc., are stated in the text.

On the crustal model suggested the rise of granite magma will be accompanied by a net upward migration of radioactive material. Consequently the steady-state temperatures in the crust will be lowered after emplacement of a granite.

8. Testing the hypothesis of a low density granite layer within the continental crust

Two main suggestions have been made. The first idea, that the top part of the upper layer is both denser and chemically more basic than granite or grano-diorite, seems to be well established. The second suggestion of an underlying truly granitic zone needs to be further tested. Two tests are suggested; both are seismological.

The first relates to the low velocity lithosphere channel proposed by Gutenberg (1951). The implication of this paper is that this low-velocity channel is directly connected to the surface through granite batholiths. If this suggestion is correct, then some of the energy normally trapped within the low-velocity layer would be

expected to emerge both directly and by diffraction through large granite masses. Thus a detailed study of the amplitude variation of Lg (particularly Lg2) on and around a large batholith might be of considerable interest.

The second test suggested is a seismic refraction study of a large granite to determine the depth of the underlying high-velocity rocks.

9. Conclusions

The main conclusion of the paper is that the upper layer of the continental crust may be subdivided into an upper metasedimentary (or dioritic) zone of intermediate composition overlying a lower granitic zone. The model of the continental crust which has been suggested is stated in tabulated form n Table 1 and illustrated in relation to the suggested origin of granite batholiths in Figure 5.

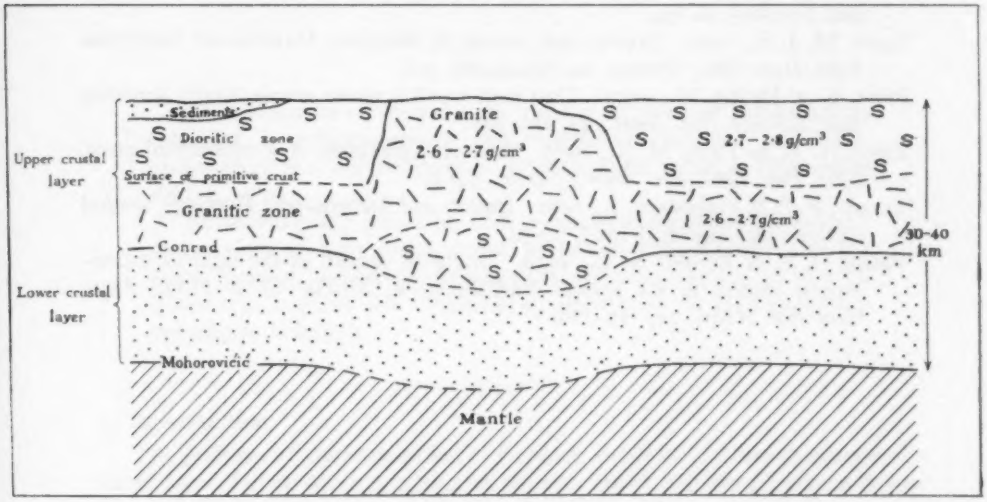

FIG. 5.—The proposed model of continental crustal structure in relation to a typical granite batholith.

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Dispersive Properties of Liquid overlying an Aeolotropic Half-space

Iya Abubakar (Part I)

and

J. A. Hudson (Part II)

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Summary

The dispersion equation for the velocity of waves of Rayleigh type travelling in a homogeneous and transversely isotropic half-space, above which is a uniform layer of liquid of finite depth, is obtained in Part I.

Phase and group velocities of the waves were calculated numerically as functions of wave number and for several different values of the elastic parameters and the dispersion curves are presented in Part II. These are compared with the curves obtained when the solid medium is completely isotropic.

Part I

1. Introduction

The problem of the propagation of Rayleigh waves in a system consisting of a liquid layer of finite depth overlying an isotropic elastic half-space has been treated by numerous investigators, notably Stoneley (1926), Biot (1952), Tolstoy (1954). This has been extended to take into account the layering of the ocean floor (Stoneley 1957). The main result found is that the wave propagation is dispersive, and this provides a probable explanation for the long unresolved problem of the "coda" as a dispersion phenomenon.

There is no direct evidence of aeolotropy of the ocean floor. But that may be due to lack of precision in seismological observations sufficient to make some departures from isotropy discernible in seismograms. It is possible that sediments when settling may be orientated in a preferential pattern instead of in random directions.

In this paper the problem of the wave propagation in a liquid overlaying a semi-infinite homogeneous aeolotropic elastic medium is treated for the case of two-dimensional unattenuated waves. The frequency equation is obtained.

2. The equations of motion

Choose a rectangular coordinate system (x, z) as shown (Figure 1) with the z-axis pointing downwards. Let the liquid, with density ρ_1 and bulk modulus λ_1, of

depth h occupy the region $-\infty < x < +\infty$, $-h \leqslant z \leqslant 0$. Underneath it is a half-space of homogeneous aeolotropic elastic material whose density is ρ_2 and whose elastic properties are defined by the condition (Love 1944) that its strain energy function has the form (in the two-dimensional case):

$$2W = Ae_{xx}^2 + Ce_{zz}^2 + 2Fe_{xx}e_{zz} + Le_{xz}^2. \tag{1}$$

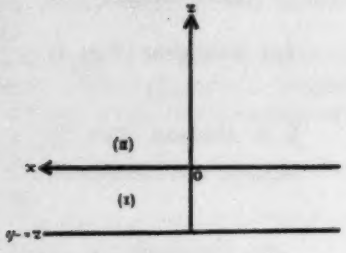

Fig. 1.—Axes of coordinates.

Since W is a positive definite form we have (Satô, 1950):

$$A > 0, \quad C > 0, \quad L > 0, \quad AC > F^2. \tag{2}$$

We shall also assume

$$A > L \quad \text{and} \quad C > L. \tag{3}$$

Then from (1) we obtain:

$$\begin{aligned} p_{zz} &= Ce_{zz} + Fe_{xx} \\ p_{xz} &= Le_{xz} \\ p_{xx} &= Ae_{xx} + Fe_{zz}. \end{aligned} \tag{4}$$

Let u, w be the displacements in the x, z directions respectively.

On making use of Love's relations, (Love 1944), between strains and displacements, e.g.

$$e_{xx} = \partial u/\partial x, \quad e_{xz} = \partial u/\partial z + \partial w/\partial x,$$

relations (4) become:

$$\begin{aligned} p_{zz} &= C\,\partial w/\partial z + F\,\partial u/\partial x \\ p_{xz} &= L(\partial u/\partial z + \partial w/\partial x) \\ p_{xx} &= A\,\partial u/\partial x + F\,\partial w/\partial z. \end{aligned} \tag{5}$$

Assuming that A, C, L, F are constant and that displacements are small, the equations of motion in such a medium are (Ewing, Jardetzky & Press 1957):

$$\begin{aligned} \rho_2 \frac{\partial^2 u}{\partial t^2} &= A\frac{\partial^2 u}{\partial x^2} + L\frac{\partial^2 u}{\partial z^2} + (F+L)\frac{\partial^2 w}{\partial x \partial z} \\ \rho_2 \frac{\partial^2 w}{\partial t^2} &= L\frac{\partial^2 w}{\partial x^2} + C\frac{\partial^2 w}{\partial z^2} + (F+L)\frac{\partial^2 u}{\partial x \partial z}. \end{aligned} \tag{6}$$

On putting

$$A = C = \lambda + 2\mu, \quad F = \lambda, \quad L = \mu \tag{7}$$

it is readily verified that the above equations (5) and (6), reduce to the familiar form for an isotropic body with Lamé's constants λ, μ

Assuming only small disturbances from the state of undisturbed equilibrium and neglecting gravity, which is reasonable for waves of the periods prevailing in earthquakes (Stoneley 1957), the equation of motion in the liquid can be expressed in terms of the displacement potential Φ_1 given by:

$$\frac{\partial^2\Phi_1}{\partial x^2} + \frac{\partial^2\Phi_1}{\partial z^2} = \frac{\rho_1}{\lambda_1}\frac{\partial^2\Phi_1}{\partial t^2} = \frac{1}{c_1^2}\frac{\partial^2\Phi_1}{\partial t^2}, \tag{8}$$

where $c_1 = (\lambda_1/\rho_1)^{\frac{1}{2}}$ is the velocity of sound in the liquid. The displacements, u_1, w_1, in the liquid are given by:

$$\begin{aligned} u_1 &= \Phi_{1x} \\ w_1 &= \Phi_{1z}. \end{aligned} \tag{9}$$

The pressure, p, in the liquid is given by:

$$p = -\rho_1\frac{\partial^2\Phi_1}{\partial t^2}. \tag{10}$$

Let the waves in the liquid have wave number k and phase velocity c in the x-direction. Then the displacement potential can be written in the form:

$$\Phi_1 = \Phi_1^{(0)}(z)\exp[ik(x-ct)]. \tag{11}$$

Inserting this in (8) yields:

$$\frac{d^2\Phi_1^{(0)}(z)}{dz^2} + k^2(c^2/c_1^2 - 1)\Phi_1^{(0)}(z) = 0,$$

from which we obtain

$$\Phi_1^{(0)}(z) = E_1\cos(k_0 z) + G_1\sin(k_0 z) \tag{12}$$

where E_1, G_1 are constants and

$$k_0^2 = k^2(c^2/c_1^2 - 1). \tag{13}$$

Hence inserting (12) into (11) yields:

$$\Phi_1 = [E_1\cos(k_0 z) + G_1\sin(k_0 z)]\exp[ik(x-ct)]. \tag{14}$$

In the solid medium let the displacements be given by:

$$\begin{aligned} u &= U(z)\exp[ik(x-ct)] \\ w &= W(z)\exp[ik(x-ct)]. \end{aligned} \tag{15}$$

Putting (15) into (6) yields:

$$\begin{aligned} L\frac{d^2U}{dz^2} + ik(F+L)\frac{dW}{dz} - (A-\rho_2 c^2)k^2U &= 0 \\ C\frac{d^2W}{dz^2} + ik(F+L)\frac{dU}{dz} - (L-\rho_2 c^2)k^2W &= 0. \end{aligned} \tag{16}$$

As it is a plane wave problem we expect an exponential dependence of the displacement on z, and from the form of (16) both U, W will have the same exponents.

Hence let

$$\begin{aligned} U &= Pe^{-ksz} \\ W &= Qe^{-ksz} \end{aligned} \tag{17}$$

where P, Q are constants. Inserting (17) into (16) yields:

$$\begin{aligned} LPs^2 - is(F+L)Q - (A-\rho_2c^2)P &= 0 \\ CQs^2 - is(F+L)P - (L-\rho_2c^2)Q &= 0. \end{aligned} \tag{18}$$

The consistency of (18) demands that the discriminant of the pair of equations be zero which yields:

$$\begin{aligned} LCs^4 + s^2[(F+L)^2 - C(A-\rho_2c^2) - L(L-\rho_2c^2)] + \\ +(A-\rho_2c^2)(L-\rho_2c^2) = 0. \end{aligned} \tag{19}$$

Using Stoneley's notation (Stoneley 1949):

$$\begin{aligned} F+L = J, \quad \rho_2c^2 - A = R, \quad \rho_2c^2 - L = S \\ J^2 + RC + LS = \Gamma, \end{aligned} \tag{20}$$

(19) becomes:

$$LCs^4 + \Gamma s^2 + RS = 0. \tag{21}$$

Let s_1^2, s_2^2 be the roots of (21). They are given by:

$$[-\Gamma \pm (\Gamma^2 - 4LCRS)^{\frac{1}{2}}]/2LC \tag{22}$$

On using relations (7) the two roots given by (22) reduce to:

$$1 - c^2/\alpha^2 \quad \text{and} \quad 1 - c^2/\beta^2$$

where α, β are the velocities of P and SV waves in the isotropic medium. This agrees with the isotropic case.

In order that the energy of these plane waves be confined to the superficial layer of the medium, we require that $Re\, s > 0$.

It is not possible to show, in the general case, that

$$\Gamma^2 - 4LCRS > 0$$

or substituting the value of Γ from (20),

$$J^4 + (RC - LS)^2 + 2J^2(RC + LS) > 0, \tag{23}$$

except for $c^2 > A/\rho_2$ when the inequality holds for all cases. For the class of isotropy, for which the elastic parameters are related by (Stoneley 1949):

$$A = C = F + 2L \tag{24}$$

the inequality (23) is true. Hence appealing to continuity we expect (23) to hold for a number of elastic media with a slight departure from isotropy. Moreover it is true for the case of beryl whose elastic constants are (Love 1944):

$$A = 26{\cdot}94, \quad C = 23{\cdot}63, \quad F = 6{\cdot}61, \quad L = 6{\cdot}53, \quad \rho_2 = 2{\cdot}7 \tag{25}$$

here the elastic constants are measured in 10^{11} dyn/cm^2.

We limit our consideration to substances for which s^2 is real. The condition $Re\, s > 0$ now requires $s^2 > 0$.

Let us write (21) in the form:

$$f(s^2) = LCs^4 + \Gamma s^2 + RS = 0. \tag{26}$$

The roots of $f(s^2) = 0$ are both positive (since $C > 0$, $L > 0$) if R and S have the same sign and $\Gamma < 0$. Since it is assumed that $A > L$, R and S are both positive if $c^2 > A/\rho_2$ and both are negative if $c^2 < L/\rho_2$. Thus for real positive roots of (26) we require that $c^2 < L/\rho_2$.

Stoneley (1949) has shown that if the elastic constants of the medium are related by (24) it transmits two body waves whose velocities are $(A/\rho_2)^{\frac{1}{2}}$ and $(L/\rho_2)^{\frac{1}{2}}$. The above analysis shows that these two velocities are significant even if the medium is not capable of propagating uncoupled body waves—that is when the relations (24) do not hold.

To see the physical significance of these two velocities let us consider the case of a plane wave propagating in the aeolotropic medium. Let k, c be its wave number and velocity respectively, and let θ be its angle of incidence on the plane boundary of the medium.

Then following Stoneley (1949) we obtain the expression for the velocity of body waves in the medium as:

$$\begin{aligned}\rho_2 c^2 = {} & [(A+L)\sin^2\theta + (L+C)\cos^2\theta \pm \\ & \pm \{[(A-L)\sin^2\theta + (L-C)\cos^2\theta]^2 + 4(F+L)^2 \sin^2\theta \cos^2\theta\}^{\frac{1}{2}}]/2.\end{aligned} \tag{27}$$

If $\theta = \pi/2$ from (27) we obtain $c = (A/\rho_2)^{\frac{1}{2}}$, $c = (L/\rho_2)^{\frac{1}{2}}$. This corresponds to propagation parallel to the x-axis.

If we put $\theta = \pi/2$ in the equations used in deriving the expression (27) for c^2 we see that $(A/\rho_2)^{\frac{1}{2}}$ is the velocity associated with the component of displacement parallel to the direction of propagation and $(L/\rho_2)^{\frac{1}{2}}$ is the velocity associated with the component of displacement perpendicular to the direction of propagation. Hence $(A/\rho_2)^{\frac{1}{2}}$ and $(L/\rho_2)^{\frac{1}{2}}$ are the velocities of the longitudinal and distortional waves respectively for propagation parallel to the x-axis. In our problem we are concerned mainly with propagation in that direction.

By putting $\theta = 0$ in (27) we readily find that the corresponding velocities for propagation parallel to the z-axis are $(C/\rho_2)^{\frac{1}{2}}$ and $(L/\rho_2)^{\frac{1}{2}}$.

Thus the solution of equations (16) which dies away with depth is:

$$\begin{aligned} U &= P_1 e^{-ks_1 z} + P_2 e^{-ks_2 z} \\ W &= Q_1 e^{-ks_1 z} + Q_2 e^{-ks_2 z} \end{aligned} \tag{28}$$

where P_j, Q_j are not independent but are related by (18). That is

$$\frac{Q_j}{P_j} = \frac{Ls_j^2 - (A - \rho_2 c^2)}{is_j(F+L)} = \frac{Ls_j^2 + R}{is_j J} = m_j \text{ say} \tag{29}$$

where $j = 1, 2$.

Thus the displacements appropriate to medium (II) are:

$$\begin{aligned} u_2 &= (E_2 e^{-ks_1 z} + G_2 e^{-ks_2 z}) \exp[ik(x - ct)] \\ w_2 &= (m_1 E_2 e^{-ks_1 z} + m_2 G_2 e^{-ks_2 z}) \exp[ik(x - ct)] \end{aligned} \tag{30}$$

where s_j, m_j are given by (22) and (32) respectively.

3. Boundary conditions

The boundary conditions are the vanishing of pressure at the free surface of the liquid, and the continuity of stresses and normal displacement at the liquid–solid interface. Thus for all x and t,

$$\begin{aligned} p &= 0 && \text{on } z = -h \\ p_{zz} &= -p && \text{on } z = 0 \\ p_{zx} &= 0 && \text{on } z = 0 \\ w_1 &= w_2 && \text{on } z = 0. \end{aligned} \tag{31}$$

Using the expressions for the pressure, stresses and displacements given by (10), (5), (9) and (25), the boundary conditions (31) give:

$$\begin{aligned} & E_1 \cos(k_0 h) - G_1 \sin(k_0 h) && = 0 \\ & \rho_1 c^2 k^2 E_1 + ikF(E_2 + G_2) + kC(-m_1 s_1 E_2 - m_2 s_2 G_2) && = 0 \\ & -ks_1 E_2 - ks_2 G_2 + ik(m_1 E_2 + m_2 G_2) && = 0 \\ & m_1 E_2 + m_2 G_2 - k_0 G_1 && = 0. \end{aligned} \tag{32}$$

Eliminating the coefficients E_1, E_2, G_1, G_2 we obtain

$$\begin{vmatrix} \cos k_0 h & -\sin k_0 h & 0 & 0 \\ \rho_1 c^2 k^2 & 0 & ikF - m_1 s_1 kC & ikF - m_2 s_2 kC \\ 0 & 0 & ikm_1 - ks_1 & ikm_2 - ks_2 \\ 0 & -k_0 & m_1 & m_2 \end{vmatrix} = 0$$

Multiplying this out yields:

$$\rho_1 c^2 \frac{k}{k_0} \tan k_0 h = \frac{(m_1 - m_2)(F + s_1 s_2 C) + i(s_1 - s_2)(F - m_1 m_2 C)}{m_1 s_2 - m_2 s_1}.$$

Inserting the values of s_1, s_2 and m_1, m_2 given by (22) and (29) respectively and rearranging we obtain:

$$\tan[kh(c^2/c_1^2 - 1)^{\frac{1}{2}}] = \frac{(F^2/R + C)(RS/LC)^{\frac{1}{2}} - \rho_2 c_2^2}{[-\Gamma/LC + 2(RS/LC)^{\frac{1}{2}}]^{\frac{1}{2}}} \cdot (c^2/c_1^2 - 1)^{\frac{1}{2}}/\rho_1 c^2. \tag{33}$$

This is the equation relating the phase velocity, c, to the wavelength, $2\pi/k$. The wavelength is a multi-valued function of the phase velocity, each value corresponding to a different mode of propagation. Given any value of k, c can be determined from (33) and subsequently the ratios of the constants can be obtained from any three of the equations (32) and hence the amplitude of the motion may be determined for all depths.

As $c^2 \to A/\rho_2$, $R \to 0$ and the right-hand side of equation (33) $\to \infty$.
Hence:

$$kh(c^2/c_1 - 1)^{\frac{1}{2}} \to (2n-1)\pi/2 \quad \text{as} \quad c^2 \to A/\rho_2$$

or

$$kh \to (n - \tfrac{1}{2})\pi(A/\rho_2 c_1^2 - 1)^{\frac{1}{2}}. \tag{34}$$

As $c \to c_1$,

$$kh \to n\pi/(c^2/c_1^2 - 1)^{\frac{1}{2}} \to \infty. \tag{35}$$

These are similar to the observations made by Ewing, Jardetzky & Press (1957, p. 139) concerning an isotropic medium.

Using the relations (7), the dispersion equation reduces to:

$$\tan[kh(c^2/c_1^2-1)^{\frac{1}{2}}] = \frac{\rho_2\beta^4}{\rho_1 c^4}(c^2/c_1^2-1)^{\frac{1}{2}}[4(1-c^2/\alpha^2)^{\frac{1}{2}}-(1-c^2/\beta^2)^{\frac{1}{2}}-(2-c^2/\beta^2)^2]/(1-c^2/\alpha^2)^{\frac{1}{2}} \quad (36)$$

which is the dispersion equation of liquid overlying a semi-infinite homogeneous isotropic elastic solid (Ewing *et al*, 1957, p. 161).

As $kh \to 0$, from (33) we have:

$$(F^2/R+C)(RS/LC)^{\frac{1}{2}}-\rho_2 c^2 \to 0.$$

The vanishing of this expression gives the velocity of Rayleigh waves in the solid medium (Stoneley 1949). Hence as the wavelength becomes very long compared to the thickness of the liquid layer the phase velocity approaches the velocity of Rayleigh waves in the solid medium.

The group velocity can be derived from the dispersion equation (33) by the usual formula.

4. Acknowledgment

The author is grateful to Dr R. Stoneley, F.R.S., whose lectures suggested the topic, and to Dr E. R. Lapwood for useful discussions.

St. Catherine's College,
Cambridge:
1960 *December.*

Part II

1. Introduction

In Part I Abubakar deduced the dispersion equation for waves of Rayleigh type travelling in an aeolotropic elastic solid above which is a uniform layer of liquid of finite depth. The solid medium is assumed to be semi-infinite in extent and to be homogeneous and transversely isotropic.

The elastic properties of the medium are defined by the form of the strain energy function, W, which is (Part I, equation (1))

$$2W = Ae_{xx}^2+Ce_{zz}^2+2Fe_{xx}e_{zz}+Le_{xz}^2 \quad (1)$$

in the two-dimensional case.

The dispersion equation connecting the wave number and phase velocity of the waves is (Part I, equation (33)).

$$\tan\left[kh\left(\frac{c^2}{c_1^2}-1\right)^{\frac{1}{2}}\right] = \frac{\left(\frac{F^2}{R}+C\right)\left(\frac{RS}{LC}\right)-\rho_2 c^2}{\left[2\left(\frac{RS}{LC}\right)^{\frac{1}{2}}-\frac{\Gamma}{LC}\right]^{\frac{1}{2}}} \cdot \frac{\left(\frac{c^2}{c_1^2}-1\right)^{\frac{1}{2}}}{\rho_1 c^2} \quad (2)$$

where c is the phase velocity and k the wave number of the waves, c_1 is the velocity

of sound in the liquid, ρ_1 and ρ_2 are the densities of the liquid and the solid respectively, and h is the depth of the liquid layer:

$$R = \rho_2 c^2 - A, \quad S = \rho_2 c^2 - L$$

and

$$\Gamma = (F+L)^2 + RC + LS.$$

For the purposes of calculation the variables were changed to

$$\left.\begin{aligned} y &= kh \\ x &= c/c_1 \end{aligned}\right\} \tag{3}$$

and the equation was written

$$\tan\left[y\sqrt{(x^2-1)}\right] = \frac{\sqrt{(x^2-1)}}{\beta x^2} \cdot \frac{\left[x^2 + \left(\frac{F^2-AC}{LC}\right)\alpha\right]\sqrt{\left[\frac{C}{L}\left(\frac{x^2-\alpha}{x^2-\alpha.(A/L)}\right)\right]} - x^2}{\left[\left(\frac{AC-F^2-2FL}{LC}\right)\alpha - x^2\left(1+\frac{L}{C}\right) + 2\sqrt{\left\{\frac{L}{C}\left(x^2-\alpha\cdot\frac{A}{L}\right)(x^2-\alpha)\right\}}\right]^{\frac{1}{2}}} \tag{4}$$

where

$$\alpha = \frac{L}{c_1^2 \rho_2} \quad \text{and} \quad \beta = \frac{\rho_1}{\rho_2}. \tag{5}$$

Numerical calculations of y in terms of x were made at the Cambridge University Mathematical Laboratory on EDSAC II. The group velocity was also calculated from the formula

$$U = c + k\frac{dc}{dk};$$

that is

$$\frac{U}{c_1} = x + y\frac{dx}{dy}. \tag{6}$$

The constants used were as far as possible typical of the practical case of an ocean bed but the only elastic constants for an aeolotropic medium which could be found were for beryl. These were therefore used and the calculations served to show the characteristics of the dispersion curves rather than useful values of phase and group velocities.

In all the calculations (Love 1944)

$$A = 26{\cdot}94, \quad C = 23{\cdot}63, \quad F = 6{\cdot}61, \quad L = 6{\cdot}53 \tag{7}$$

measured in 10^{11} dyn/cm^2.

Abubakar showed in Part I (equation (27)) that in an aeolotropic solid medium, whose properties are defined by equation (1), longitudinal and distortional waves travel with the respective velocities $(A/\rho_2)^{\frac{1}{2}}$ and $(L/\rho_2)^{\frac{1}{2}}$ parallel to the x-axis and $(C/\rho_2)^{\frac{1}{2}}$ and $(L/\rho_2)^{\frac{1}{2}}$ parallel to the z-axis. It follows that the effect of the aeolotropy of a solid medium is revealed in the difference of the velocities of longitudinal waves in the two directions. The ratio of the two is $(A/C)^{\frac{1}{2}}$.

Using the constants given by (7) the value is

$$(A/C)^{\frac{1}{2}} = 1{\cdot}07.$$

This value compares fairly well with the value of 1·09 found by Cholet & Richard (1954) for the vertical anisotropy of Cretaceous sands and shales in the Sahara region. Ricker (1951) found that the horizontal velocities of waves in Cretaceous shales in Colorado are 15–18 per cent higher than the vertical velocities.

2. The first mode

Figure 1 shows the curves of the phase and group velocities as they vary with wave number in the first mode. These appear to be similar in every respect to the curves drawn by Tolstoy (1954, Figures 5 and 9) for the completely isotropic case.

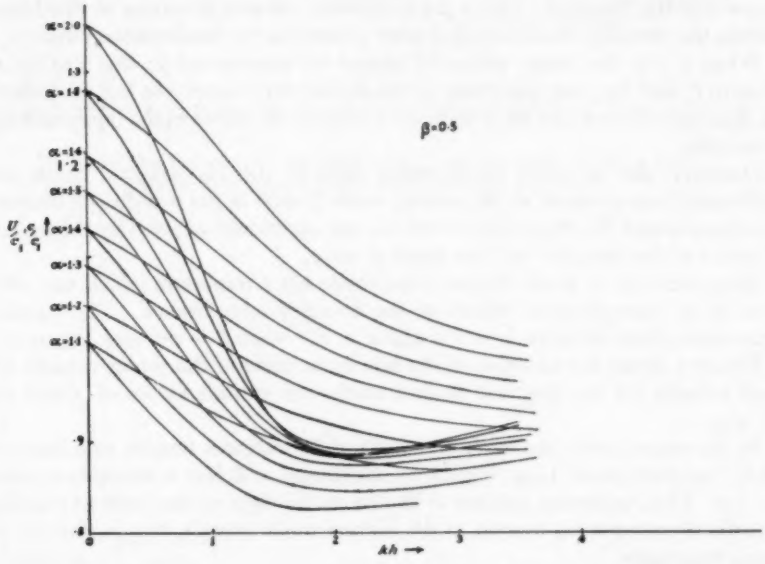

FIG. 1.—The variation of the phase velocity (the upper curve in each pair) and group velocity (the lower curve) with the wave number for various values of α.

As $kh \to 0$ both U and c tend to the same value c_R. c_R is the relevant solution of

$$\left[\frac{F^2}{\rho_2 c^2 - A} + C\right]\left[\frac{(\rho_2 c^2 - A)(\rho_2 c^2 - L)}{LC}\right]^{\frac{1}{2}} - \rho_2 c^2 = 0 \qquad (8)$$

and is the velocity of Rayleigh waves in the solid medium with a free upper surface (Stoneley 1949). The result is understandable physically since $kh \to 0$ means that the waves have wavelengths large compared with the depth of the liquid layer and thus the effect of the layer is negligible.

As $kh \to \infty$, that is the wavelength becomes small compared with the depth of the layer, the phase and group velocities tend to the common value c_s, the velocity

of sound in the liquid, ρ_1 and ρ_2 are the densities of the liquid and the solid respectively, and h is the depth of the liquid layer:

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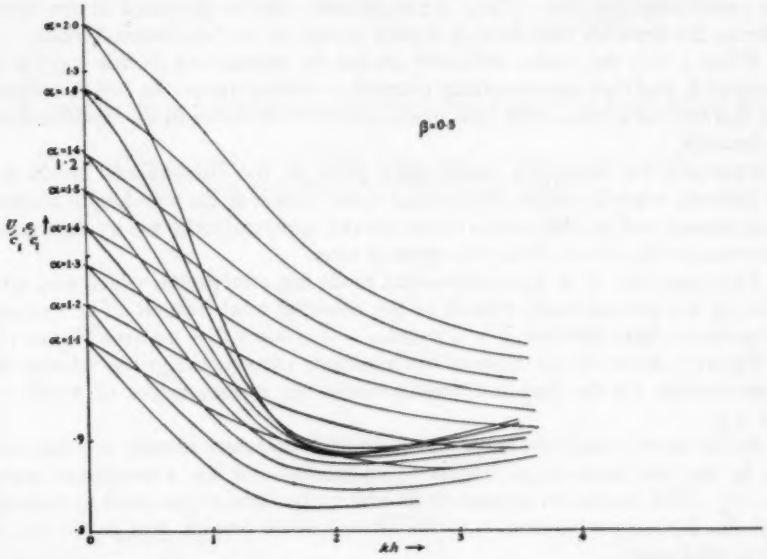

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As $kh \to 0$ both U and c tend to the same value c_R. c_R is the relevant solution of

$$\left[\frac{F^2}{\rho_2 c^2 - A} + C\right]\left[\frac{(\rho_2 c^2 - A)(\rho_2 c^2 - L)}{LC}\right]^{\frac{1}{2}} - \rho_2 c^2 = 0 \qquad (8)$$

and is the velocity of Rayleigh waves in the solid medium with a free upper surface (Stoneley 1949). The result is understandable physically since $kh \to 0$ means that the waves have wavelengths large compared with the depth of the liquid layer and thus the effect of the layer is negligible.

As $kh \to \infty$, that is the wavelength becomes small compared with the depth of the layer, the phase and group velocities tend to the common value c_s, the velocity

of Stoneley waves at the interface of solid and liquid when the liquid is semi-infinite in extent. c_s/c_1 is the solution of

$$\frac{\sqrt{(1-x^2)}}{\beta x^2}\left\{\left[x^2+\alpha\left(\frac{F^2-AC}{LC}\right)\right]\sqrt{\left[\frac{C}{L}\left(\frac{x^2-\alpha}{x^2-\alpha\,.\,(A/L)}\right)\right]}-x^2\right\}$$
$$=\left\{\alpha\left(\frac{AC-F^2-2FL}{LC}\right)-x^2\left(1+\frac{L}{C}\right)+2\sqrt{\left[\frac{L}{C}\left(x^2-\alpha\cdot\frac{A}{L}\right)(x^2-\alpha)\right]}\right\}^{\frac{1}{2}} \qquad (9)$$

(which is the condition that the right-hand side of equation (4) equals $\sqrt{-1}$).

The part of the curve where $c > c_1$ is called the normal-mode branch and the part where $c < c_1$ the Stoneley-wave branch.

Both Tolstoy (1954) and Biot (1952) have investigated the nature of the waves in each branch when the solid medium is isotropic. The waves of the normal mode branch may be represented as the total reflection of plane waves within the liquid layer. The phase velocity c is given by $c_1/\sin\theta$ where θ is the angle of incidence of the wave at the interface. There are an infinite number of modes in this branch, whereas the Stoneley wave branch is only present in the fundamental mode.

When $c < c_1$ the waves evidently cannot be represented in this way for real values of θ, and they are essentially coupled boundary waves. In fact Biot showed that this type of wave may be present even when body waves in the liquid and solid do not exist.

However, the transition which takes place in the fundamental mode from the Stoneley wave branch to the normal mode branch as the wavelength decreases is continuous and the dispersion curves are not apparently affected by the essential difference in the nature of the two types of wave.

Each curve for U in the fundamental mode has a minimum which may either occur in the normal-mode branch or the Stoneley-wave branch. The transition occurs somewhere between $\alpha = 1{\cdot}5$ and $\alpha = 1{\cdot}6$ when $\beta = 0{\cdot}5$ (see Figure 1).

Figure 3 shows the variation of the minimum value of the group velocity with phase velocity for the first and second modes for various values of α and with $\beta = 0{\cdot}3$.

In the second mode the minimum value of U decreases steadily as α increases, but in the first mode $U_{\min}$ begins by increasing and has a maximum around $\alpha = 1{\cdot}5$. This maximum appears to lie just to the right of the point of transition from the Stoneley-wave branch to the normal-mode branch, that is, for c/c_1 just greater than unity.

3. The higher modes

An infinite number of modes higher than the fundamental exists for $L/\rho_2 > c^2$, $c > c_1$ (so long as $c_1^2 < L/\rho_2$, otherwise no higher modes exist).

Curves for the first five modes for $\alpha = 2{\cdot}0$, $\beta = 0{\cdot}3$ are drawn in Figure 4 and again they are quite similar to the isotropic case (Tolstoy, 1954, Figure 4).

As the wavelength becomes small compared with the depth of the layer, $kh \to \infty$ and the phase and group velocities of every mode tend to the common value c_1, the velocity of compressional waves in the liquid.

On the other hand, both U and c tend to the value $(L/\rho_2)^{\frac{1}{2}}$ as the wavelength approaches a finite maximum cut-off value. This cut-off value of wavelength is smaller for each successive mode. The velocity $(L/\rho_2)^{\frac{1}{2}}$ is that of distortional waves in the solid travelling horizontally or vertically (Abubakar, Part I).

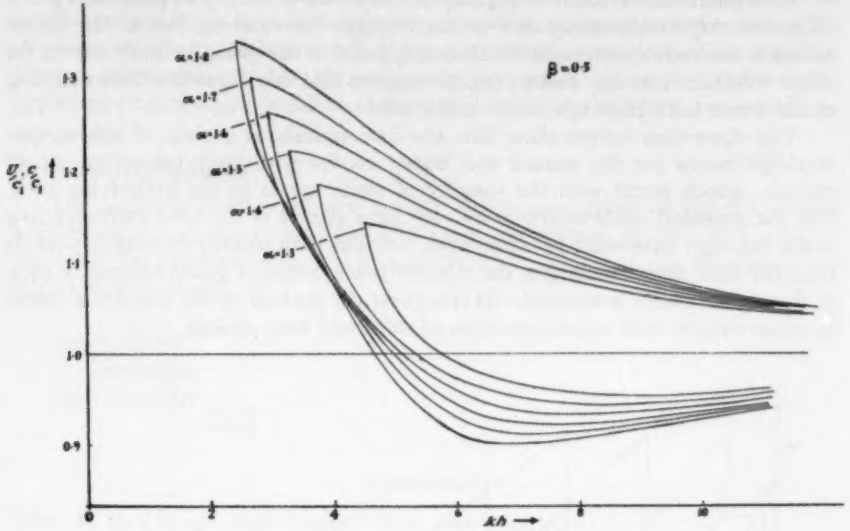

FIG. 2.—The variation of the phase and group velocities of the second mode with wave number for various values of α.

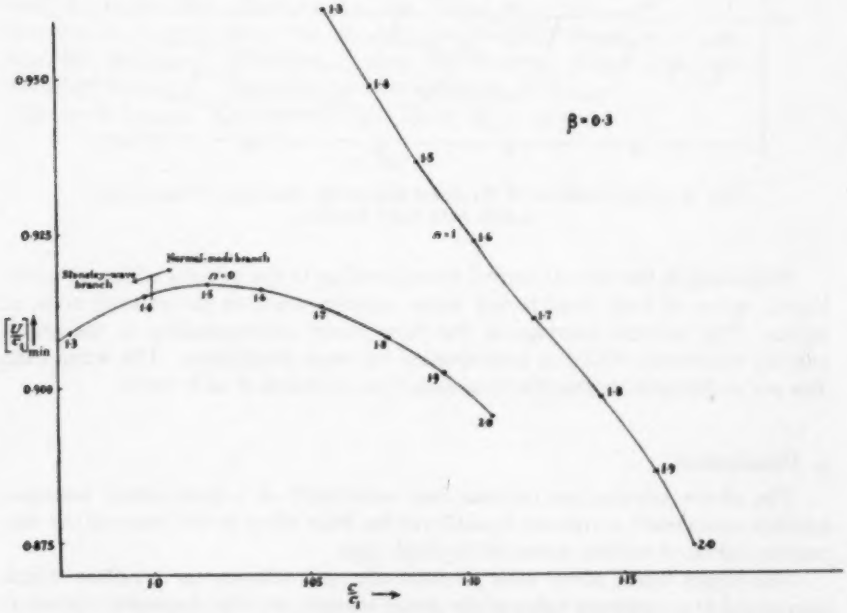

FIG. 3.—The variation of the group velocity minimum in the first and second modes with α.

Each mode has a minimum of group velocity with $U < c_1$. The phase and group velocities vary continuously as c passes through the value c_R but in the higher modes a successive minimum–maximum appears in the group velocity curves for phase velocities near c_R. Tolstoy (1954) suggests that this is due to a loose coupling of the waves with Rayleigh waves in the solid.

The dispersion curves show that the first arrivals in a train of sub-oceanic Rayleigh waves are the second and higher modes with their respective cut-off periods, which travel with the velocity of shear waves in the underlying rock. The fundamental mode arrives with very large period at the time corresponding to the Rayleigh wave velocity. The waves continue with steadily decreasing periods until the time corresponding to the minimum–maximum of group velocity of each of the higher modes is reached. At this point the periods of the respective waves decrease sharply with superimposition of short and long periods.

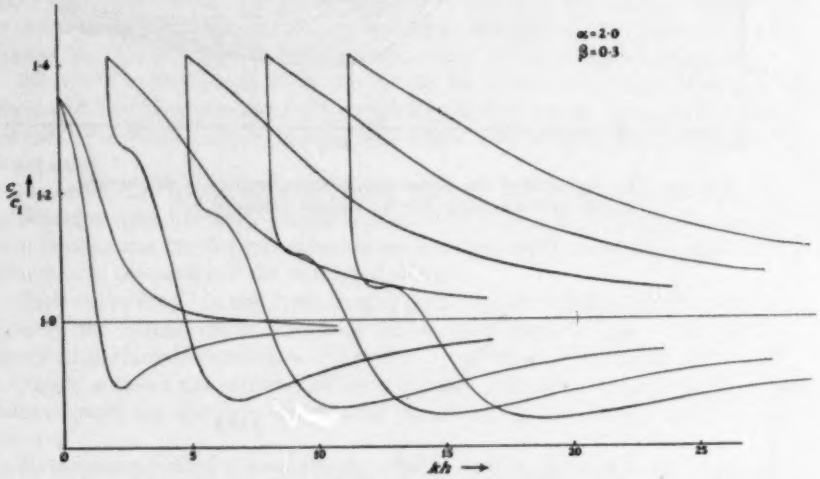

FIG. 4.—The variation of the phase and group velocities of the first five modes with wave number.

Beginning at the time of arrival corresponding to the velocity of sound in the liquid, waves of very short period arrive superimposed on the existing train of waves. The periods converge at the Airy phase, corresponding to the group velocity minimum, which is accompanied by large amplitudes. The wave train dies out as the time reaches the minimum time of arrival of each mode.

4. Conclusion

The above calculations indicate that aeolotropy of a transversely isotropic medium underneath a uniform liquid layer has little effect on the shape of the dispersion curves of surface waves of Rayleigh type.

The phases which occur most prominently on a seismogram are those which correspond to a stationary value of the group velocity, and the dispersion curves of this paper show the same maxima and minima as Tolstoy found in the completely sotropic case.

Thus it would be difficult to detect aeolotropy in the ocean bed from the dispersion curves of this type at the present time when no experimental values of the elastic constants of aeolotropic rocks liable to be found on the sea bed are available. Further calculations of this sort using such data would give arrival times of the important phases which could be directly checked from the seismogram readings.

5. Acknowledgments

The author's thanks are due to Mr Swinnerton-Dyer of the Cambridge Mathematical Laboratory for help with the computations, to Mrs E. N. Mutch for doing the programming, and also to Dr E. R. Lapwood for suggesting and supervising the work.

Jesus College,
Cambridge:
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Remanent Magnetism of Carboniferous Limestone

R. L. DuBois

(Received 1961 March 27)*

Summary

Paleomagnetic data are lacking on Carboniferous limestones of England and during an investigation of iron ores the author found specimens of these limestones that could be measured. An average declination of S 41 W and inclination of minus 29 degrees was determined for the remanent magnetization. This gives a pole position of north 40 degrees latitude and 122 degrees east longitude. Intensity values of remanent magnetization ranged upwards to $27{\cdot}5 \times 10^{-6}$ emu/cm^3.

1. Introduction

During the course of a study of the magnetism of rocks of ore deposits and associated areas, the author measured the remanent magnetism of a series of specimens of British hematitic iron ore and contiguous Carboniferous limestone. This paper pertains to the remanent magnetism of the Carboniferous limestone; results on the hematites will be forthcoming.

The specimens were collected during the winter of 1959–60, from an area immediately southwest of the Lake District of England. Specifically they came from sites located at N 54°, W 3°. These locations are on either the Barrow-in-Furness or Keswick one inch maps of the Ordnance Survey of Great Britain.

The field orientation of the specimens was determined with the aid of a Brunton compass after it had been determined that there were no local anomalous magnetic effects that would effect the orientation of the compass needle. From the oriented specimens oriented discs were obtained that measured three-fourths of an inch in diameter and five-eights of an inch in length. A total of 52 disks were prepared from 19 specimens and of this total 22 disks could be measured from 14 specimens. Corrections for geological dip were made by coring normal to bedding surfaces of specimens collected at one site where dips of 8° were measured. At other sites the beds were considered to be flat lying.

The specimens measured are fine- to medium-grained, grey to reddish brown, limestone of Carboniferous age (Smith 1919). The rocks have been locally altered and even more locally have been metasomatically replaced and openings filled by hematite.

2. Experimental and calculated results

Magnetic measurements were made on the prepared disks by the methods described by Collinson, & others (1957). For the size of specimens used and for the

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conditions of instrumentation, a lower limit of reliable measurement is considered to be $2 . 10^{-7}$ emu/cm³. Results are given in Table 1 and are plotted in Figures 1 to 3. In the table, under the heading "declination," the azimuth of the north seeking pole is noted under "N pole." Its inclination is given and a minus sign means that the angle is measured above the horizontal. To determine the general direction, an average was taken of the southwest data and a second one of the northeast data by the use of direction cosines. In this treatment, individual disks are weighted equally in the overall result and the final average reading is influenced to a greater degree by those sites and samples upon which more readings were made. According to this method of calculation, an average declination of S 41W and an inclination of $-29°$ is obtained from the southwest data. Similarly for the northeast data a value of N 34 E is obtained for the declination and the value for inclination is $+53°$. A pole position for the southwest data is calculated at N 40°, E 122°. One is not calculated for the northeast data because of its lack of significance.

Table 1

Specific Gravity	Intensity 10^{-6} emu per cm³	Intensity 10^{-6} emu per gm	Susceptibility 10^{-6} emu per cm³	Susceptibility 10^{-6} emu per gm	Orientation Declination N Pole	Orientation Inclination (− up deg.)
2·39	0·8	·3	1	1	S59W	−51
2·56	1·4	·6	1	1	S72W	−44
2·48	21·2	8·6	3	1·2	S24W	−30
2·54	9·1	3·6	7·5	3·0	S35W	−35
2·59	18·6	7·2	4	1·5	S25W	−28
2·62	5·9	2·3	5	1·9	S26W	−23
2·54	7·4	2·9	4	1·6	S15W	−3
2·61	15·6	6·0	5·5	2·1	S10W	−11
2·62	27·5	10·5	5·5	2·1	S41W	−38
2·64	6·4	2·4	8·5	3·2	S44W	−31
2·64	20·7	7·8	4·5	1·7	S42W	−38
2·62	11·6	4·4	5·5	2·1	S48W	−34
2·53	24·0	9·5	5·5	2·2	S24W	−33
2·61	5·8	2·4	5·5	2·1	S45W	−29
2·56	7·7	3·0	7	2·7	N25E	+56
2·59	1·4	·5	1·5	1	N26E	+57
2·63	1·4	·5	1·5	1	N28E	+46
2·64	1·2	·4	1	1	N49E	+54
2·51	0·9	·4	1	1	S65W	−19
2·66	0·4	·2	1	1	S66W	−32
2·59	0·9	·3	1	1	S60W	−4
2·58	1	·4	1	1	S66W	−17

A measure of the uncertainties of the averages for the southwest data is given by applying Fisher's (1953) best estimate of the precision parameter κ. Calculations give a value for κ of 18·9. Determinations of cone of confidence for α_{95} give values of 7·6°.

3. Discussion

The discussion of the magnetic data as set forth in this paper is limited to simple presentation and conclusion. The reader is referred to the excellent review

article by Cox & Doell (1960) on the subject of paleomagnetism for a fuller treatment and for a discussion of pertinent theoretical aspects. The results of the magnetic measurements as given in Table 1 show considerable uniformity. Specific gravity values range from 2·39 to 2·66, and all are below the theoretical 2·72 value for calcite. Intensities of remanent magnetism range from values too weak to measure up to $27{\cdot}4 \times 10^{-6}$ emu/cm^3. A comparison of specific gravity with magnetic intensity as plotted in Figure 1 shows only a general correlation. This suggests that for ranges of low concentrations, variable amounts of ferro-magnetic material

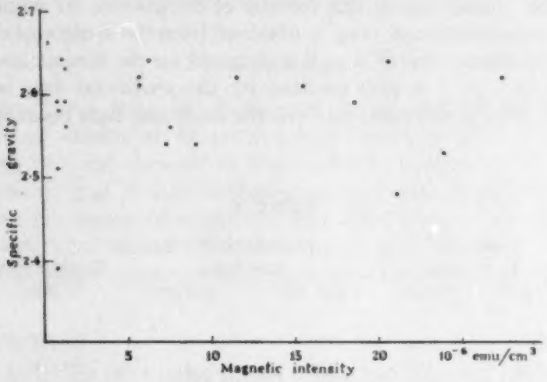

FIG. 1.—Specific gravity vs. magnetic intensity.

will have greater effects on magnetic properties than on specific gravity. Also for these ranges of concentration, individual characteristics, shape, grain size, composition, etc., will have a greater influence on intensity.

The results of susceptibility measurements given in Table 1 range from values too small to measure up to $8{\cdot}5 \times 10^{-6}$ emu/cm^3. These are related to values of intensity of remanent magnetism in Figure 2. These two variables are broadly related for the rocks studied.

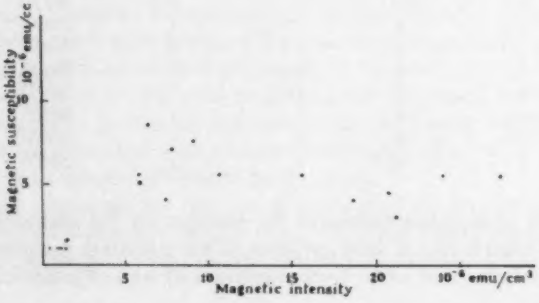

FIG. 2.—Magnetic susceptibility vs. magnetic intensity.

The orientation of the remanent magnetization is given in Table 1 and plotted in Figure 3. Readings taken on disks from the same specimen are connected together by a full line. Two distinct and separate groupings of orientatious

are obtained, a minor one to the northeast with a downward plunge, and a major one to the southwest with an upward plunge. The major direction is considered a normally magnetized direction resulting from a chemical magnetization associated with hematite formation. That it is a result of chemical magnetization is suggested by geological evidence which includes a complete lack of post-depositional thermal effects in the measured and associated rocks and also includes considerations of the overall geological environment of these rocks during and since formation.

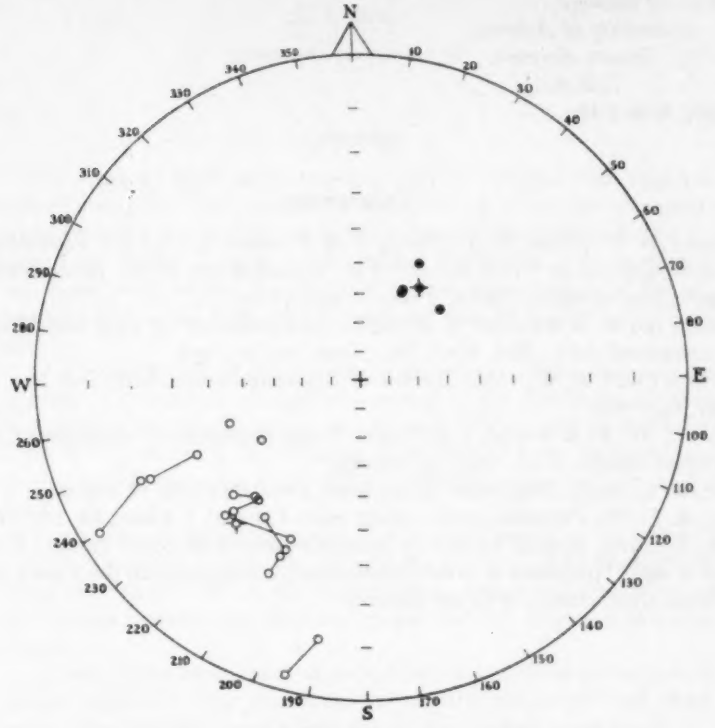

FIG. 3.—Direction of remanent magnetism.

Because of the distribution and position of the magnetic orientations, the magnetization is considered to have essentially no component of isothermal remanent magnetization. The minor direction is considered a reversed direction, the result of some self-reversal mechanism or of a magnetization during a reversed Earth's field. Everitt & Belshé (1960) concluded this latter case for some of their rocks and further concluded that the Earth's field has undergone a series of reversals during early Carboniferous times and to have finally remained reversed for a long period of time.

The magnetic orientations measured agree in part with data of Nairn (1960). The calculated paleomagnetic pole positions agree favourably with those published by Collinson & Runcorn (1960) and by Cox & Doell (1960). This agreement is not exact and of course some deviation occurs.

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Dept. of Geology,
University of Arizona,
Tucson, Arizona,
U.S.A.:
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The Effect of Water Currents on the Magnetic Remanence and Anisotropy of Susceptibility of Some Sediments

A. I. Rees

(Received 1961 January 23)

Summary

This work is a study of the magnetic properties of fine silts deposited in the laboratory. The results are described of a number of experiments in which a naturally occurring sediment was redeposited in an open flume under a range of controlled conditions. Measurements have been made of the directions of remanent magnetism of the redeposited sediments and of their principal axes of magnetic susceptibility.

It is found that the deviations of the remanence direction and of the direction of maximum susceptibility caused by the movement of water over the bed during deposition can be accounted for by an extension of the theory in which the sediment is considered as an assemblage of quasi-spherical particles which roll on deposition into hollows in the bed.

In the interpretation put forward it is postulated that the effect of a water current is to rotate the particles through an angle ρ immediately before they touch bottom, the magnitude of ρ being determined by an equilibrium between the shearing couple and the magnetic restoring couple.

A method has been found of correcting for the effects of currents in natural sediments from measurements of their remanence and anisotropy of susceptibility, and a field test of this method is suggested.

1. Introduction

The earliest work on the magnetism of the Swedish varves is that of Ising (1942) who established that the varves had an easily measureable remanent magnetism and that they had an excess of susceptibility in a plane close to the plane of stratification with a maximum direction lying in the plane of excess susceptibility.

The first investigations made in this Department were on a number of natural varve sequences from central Sweden (Griffiths 1955) and on artificial sediments produced by redepositing natural material in still water in the laboratory (King 1955). This work led to the definition of the "inclination error" and the "bedding error".

When a sediment is deposited on a flat bed it is found that I_0, the inclination of the remanence to the horizontal, is always less than I_F, that of the ambient

field, the relationship between these two quantities being

$$\tan I_0 = f \tan I_F. \tag{1}$$

f is a constant of value about 0·4. The difference

$$\delta = I_F - I_0 \tag{2}$$

has been defined as the "inclination error".

When deposition takes place on a sloping bed a further deviation of the remanence vector is found. The angle of this deviation, which varies only slightly with field inclination, is defined as the bedding error β and is usually of about the same magnitude as the slope of the bed and in the direction which would be expected were the magnetized particles to roll through a small angle down slope on deposition.

The inclination and bedding errors are accounted for by considering the rolling which takes place when a population of quasi-spherical particles falls on a bed composed of similar particles (Griffiths & others 1960). If particles are supposed to roll through an average angle ϕ, the rolling being randomly distributed over all horizontal axes, then the resultant inclination I_0 of particles originally aligned in the field is given by

$$\tan I_0 = \frac{2\cos\phi}{1+\cos\phi} \tan I_F \tag{3}$$

which, if ϕ is assumed to be constant for all field inclinations reduces to equation (1). ϕ is found to have a value of about 70° for the maximum observed value of $\delta = 28°$.

The extension of this model to the more general case of particles falling on a sloping bed is tedious, and so a simpler model has been used of four particles rolling about mutually perpendicular horizontal axes. This model predicts qualitatively the form taken by the bedding error, although the apparent variability of the bedding error under natural and laboratory conditions makes exact evaluation of the constants difficult.

It has been found that the magnitude of the inclination error found from any experiment varies inversely with the scatter σ of the remanence directions in that experiment, the quantity σ being calculated as a standard deviation using the angles between individual remanence directions and the mean direction as if they were ordinary numerical results. An approximation for this variation has been given together with suggestions as to the origin of this effect. A more accurate form of this expression based on more recent results has now been calculated. This is:

$$\delta = 22{\cdot}2° - 1{\cdot}64\sigma° \tag{4}$$

for a field inclination of 67°.

The results have been given (*ibid*) of a first series of depositions from running water and a third systematic deviation of the remanence direction—named the rotation angle ρ—has been noted. This deviation appears to be a rotation of the remanence about a horizontal axis at right angles to the direction of flow, the deviation being similar to that which would be found if, instead of being from flowing water, the deposition were on a bed dipping in the downstream direction. Lack of knowledge of conditions very near to a stream bed had, up to the present,

precluded any explanation of the form of the variation, though a simple extension of the bedding error theory predicts the direction of the rotation.

To determine accurately the field direction at the time of deposition of a sediment it is necessary to be able to make allowance for all the deviations described. It is considered that sufficient is now known to allow for the inclination and bedding errors, and the purpose of this investigation has been to find a method of correcting for the effects of the shearing stress produced by water current.

2. Laboratory sedimentation experiments

The main part of this work consisted of a series of experiments in which a varved silt from Sweden was deposited from running water in the geomagnetic field. Two experiments have been carried out in fields at shallower inclinations.

(a) *The sediment*

The sediment used was taken from a bulk sample obtained from a brick-pit at Hedemora in central Sweden, all the material being obtained from a sequence about 10 years in length dated at −700 (de Geer).

Like all the Swedish varves which have been studied for palaeomagnetic purposes the sediments are of a fine silt grade having a mean diameter of about 10 μm, and consist of almost unweathered material, derived during the latter stages of the last glaciation from the crystalline rocks of the Norwegian and Swedish Highlands. They are composed almost entirely of unweathered quartz and felspar with a small quantity of accessory minerals, the most important of which from the magnetic point of view are titanomagnetites which occur largely as inclusions in grains of mixed mineral composition.

(b) *The apparatus*

For the purpose of these studies a new flume has been constructed (Figure 1). This flume, which is of perspex mounted in a wooden framework is three metres long, 22 cm wide at its narrow end and 44 cm at its widest. It is open at the top, and the depth of water employed can be varied.

In two deposition experiments where fields other than the Earth's field were applied the small flume described in an earlier paper (Griffiths & others 1960) was used.

Measurements were made, on the samples taken from the deposition experiments, of magnetic remanence, and anisotropy of susceptibility.

Measurements of remanence were carried out on the Birmingham induction magnetometer. This instrument has been described fully elsewhere (Griffiths 1955).

The internal consistency of the measurements made on these specimens suggests an error in direction of $\frac{1}{2}^\circ$ or less. Measurements of intensity are not thought to be consistent over long periods of time but are consistent to within ± 5 per cent over the time taken to measure the specimens from one experiment.

Measurements of anisotropy of susceptibility were made using a simple suspended sample torsion magnetometer similar to that described by Ising (1942) with applied alternating fields of 35 oe A.C. Computation of the results was programmed for the Manchester University computer. No determination has been made of the errors of measurement but the consistency found between specimens from the same experiment suggests that these are not important. Since the

torsion magnetometer measures only differences in susceptibility and no instrument was available capable of measuring accurately the absolute value of the mean susceptibility this was not determined, though it is known to be of the order of 4×10^{-5} e.m.u./cm^3. So far as is known there is no significant variation of mean susceptibility from experiment to experiment.

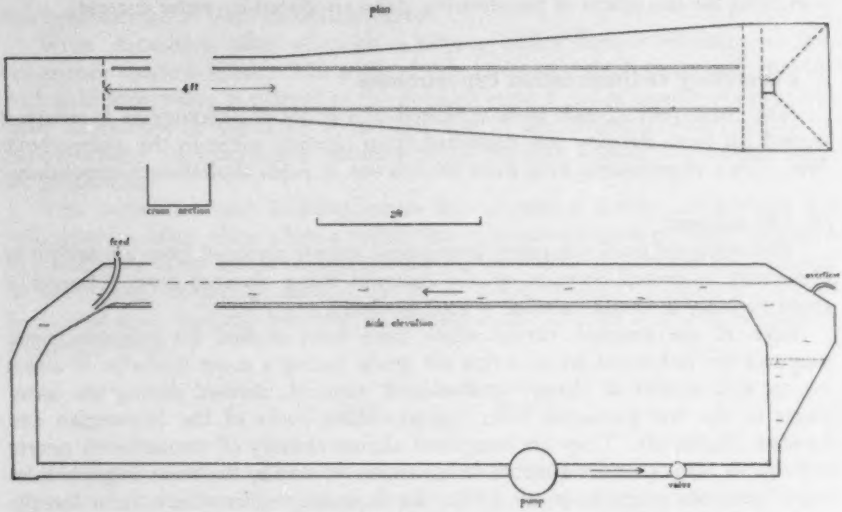

FIG. 1. The flume used in the main series of deposition experiments.

(c) *The flume experiments*

Nine flume depositions were made, seven in the new flume with flow at speeds of 1–16 cm s^{-1} and from various azimuths and two with almost still water in the old one. For each deposition a new batch of sediment of about 10–20 kg dry weight was used to form a deposit about 1 cm thick on the floor of the flume. Since the sediment was all collected from a very small volume of natural varve material there is no reason to suppose that the samples used were not effectively identical.

The conditions under which deposition takes place are of primary importance in this type of experiment for if the results are to be capable of application to the magnetism of natural sediments it is necessary to establish that the laboratory conditions are essentially similar to those in nature. The principal characteristics of natural deposition of fine silts are as follows:

(i) that the regime is entirely viscous in character (Inman 1949), that is that the layer of fluid immediately above the bed is flowing in a laminar manner and that particles within this layer are acted on only by viscous forces and obey Stokes's Law.

(ii) that deposition is slow enough for each particle to have time to complete the settling process without interference from other falling particles.

Provided that the second condition is fulfilled the fulfilment of the first is, for sediments of the sizes concerned here, quite automatic. Under certain conditions it is possible for a layer of suspension to be formed immediately above the bed in which the concentration of particles is high enough for a serious retardation

of settling to occur (Bagnold 1956). This will seriously affect the settling regime and is to be avoided if possible. With this in mind addition of sediment was made slowly, a 200 g feed taking about 5 minutes and an hour being allowed between feeds. This meant that the deposition of 1 cm of sediment took from 6 to 8 days, and with the time allowed for a very slow drainage and drying each deposition lasted about 3 weeks.

Specimens were taken by inserting a coring tube mounted above the flume, about 15 specimens being taken in 3 groups of 5 when the sediment was dry enough to make distortion in the sampling process unlikely. Since water velocity varied with position in the flume each group of 5 specimens represented a different velocity and so each deposition was made to yield 3 experiments.

The remanence directions of all specimens were measured immediately after sampling, and then the two or three specimens whose remanences were nearest the centre of each group were measured for anisotropy of susceptibility.

The experiments in the small flume were carried out to find the variation in the directions of the susceptibility axes with field inclination. Here the flume was arranged in the centre of a Helmholtz pair arranged to give fields of the same strength as that of the Earth but dipping at 20° and 40° respectively. 5 samples were taken from each experiment and all were measured for remanence and anisotropy.

3. Tangential stress in fluid flow

When describing the conditions in these experiments and when comparing them with conditions in nature it is necessary to have some parameter to describe the strength of the flow involved. A parameter which is frequently used is the flow velocity, but this is not altogether satisfactory.

In a turbulent stream the velocity varies from place to place and from time to time. This leads to a difficulty in using velocity as a measure of the force acting on particles near to the bed. The situation is further complicated by the fact that as well as depending on velocity the force depends on the dimensions of the stream and the viscosity of the suspension of particles of which the stream consists. In order to describe experimental conditions it is more convenient to work in terms of the tangential stress which is applied to the system by the action of gravity in propelling the stream.

The concept of tangential stress in fluid flow over a grain bed has been discussed by Bagnold (1956) who suggests that the measurement of stress should be in terms of a standard unit of stress related to the ultimate shear resistance of the bed. This unit he defines as

$$\gamma = (d_g - d_f)gD\cos\beta \tag{5}$$

where d_g and d_f are the grain and fluid densities, g the acceleration due to gravity, D the grain diameter, and β the downstream slope of the bed.

For parallel flow in an open channel of infinite width the tangential shearing stress $\mathscr{T}_F$ at the bed due to the fluid is

$$\mathscr{T}_F = d_f g y \sin\beta \tag{6}$$

where y is the stream depth, and its dimensionless measure, the dimensionless fluid stress, Θ_F is

$$\Theta_F = \frac{\mathscr{T}_F}{\gamma} = \frac{y d_f \tan\beta}{(d_g - d_f)D}. \tag{7}$$

The actual total stress at the bed, Θ, will depend also on the grain concentration in the fluid, but the concept of a dimensionless stress will be used here first of all in terms of the fluid stress for parallel flow. The total applied fluid stress provided by the flowing fluid can be found by measuring the slope of the surface of the fluid flowing downstream. This can be done by direct measurement but in order that sufficient accuracy can be obtained a lengthy series of measurements must be made. In view of the possibility of magnetic disturbance it was thought advisable to avoid these measurements during the actual experiments, and so instead a preliminary series of experiments was carried out in which the slope of the water surface was calibrated against the mean downstream velocity of the fastest moving part of the turbulent fluid as measured with a dye in the fluid. A measurement of the velocity was made immediately prior to each deposition by means of an injection of dye. As will be seen later the results of the measurements suggest that the accuracy achieved has been adequate.

4. Results of the magnetic measurements

(a) *The remanence measurements*

The results of the remanence measurements, which are given in Table 1, are generally consistent with those reported previously, though, presumably due to better experimental technique, there is less scatter between specimens. The variation of the inclination error noted previously is absent here and this supports the conclusion reached earlier that there is not necessarily a variation of δ with water velocity.

Table 1

Results of the remanence measurements

Flow from	Θ_F	mean D deg	mean I deg	σ deg	δ deg	ρ deg
45°	1·03	8·5	41·5	2·9	21·5	6
45	0·81	13·5	47	3·7	12	11
45	0·78	12·5	42	2·1	18	9·5
45	0·65	11·5	49	1·9	14	7·5
45	0·55	12·5	44	1·8	16	10
45	0·49	13	47	1·6	12·5	10·5
90	2·0	17	43·5	3·5	22·5	7·5
90	1·36	17·5	45	2·4	21	7·5
90	0·98	14	45	2·7	21	6
90	0·61	15	47	2·7	19	6·5
90	0·40	14	48	2·1	18	6·5
90	0·26	12	48	2·1	18·5	5
90	0·02	13·5	44	2·2	23	5·5
90	0·01	8	45·5	2·9	21·5	3
90	0·0	3	44	3·25	23	1·5
145	1·18	31	64	3·0	12·5	13·5
145	0·81	34	65	3·2	12	15
145	0·65	30	65	1·9	10	13·5
145	0·30	26	59	1·3	16	11·5
145	0·20	18·5	59·5	1·8	14·5	9
145	0·12	12	58·5	3·0	13·5	7

When deposition took place from slowly moving water it was found that the resulting sediment had horizontal bedding and a mirror smooth top surface, but

at values of Θ_F greater than 0·5, in this case representing a mean velocity of about 5 cm s^{-1}, the top surface consisted of a series of well developed ripples the structure of which was repeated in the body of the sediment. This effect, which appeared to be related to the strength of the flow, will be described in full elsewhere, but it should be mentioned that no discontinuous change in the magnetic properties of the sediment occurred at the inception of these ripples, nor did there appear to be any resulting variation in the magnetic properties of specimens taken from different parts of the same exposed ripple

It is now believed that the rotation ρ occurs before the particles touch the bed and before the process in which the inclination error is brought about. This leads to a different definition of ρ from that previously used (Figure 2) which, since the

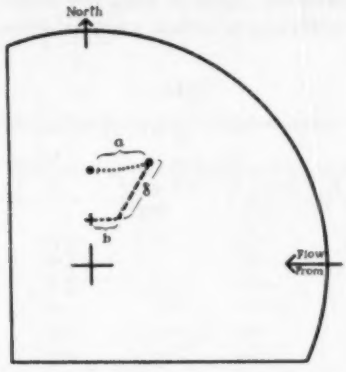

FIG. 2. Stereogram showing definitions of the angle ρ, for particle rotation occurring (a) after and (b) before touch down on the bed, and of the angle δ

effect of the variability in δ with inclination is removed from the computation of ρ, leads to a considerable improvement in the consistency of the results.

Figure 3 shows the variation of ρ with fluid stress for water flowing from 3 azimuths. The variability of the results is very much greater for flow from 45° than from the other two azimuths and for this reason it has been necessary to exclude these results from the quantitative treatment given below.

It appears that ρ varies with Θ_F and with the azimuth of flow, the greatest values observed being for flow from a magnetic bearing of 145°. As the speed of the flow is increased the value of ρ rises rapidly until the fluid stress Θ_F is about 0·4 and then more slowly to the maximum of the observed stress range. The observed points on the curves for azimuths of 90° and 145° appear to be on a curve of the form $\rho = A\Theta_F{}^n$. A least squares fit of $\log\rho/\log\Theta_F$ gives values of $n = 0{\cdot}21 \pm 0{\cdot}16$ and $n = 0{\cdot}31 \pm 0{\cdot}05$ and values of A of 7 and 15 for the 90° and 145° azimuths respectively. There is no significant difference in the values of n in the two cases and it may be assumed that both variations are of the same form. There is no correlation between ρ and σ or ρ and δ.

(b) *The measurements of anisotropy of susceptibility*

The computation of the anisotropy data defines a triaxial ellipsoid of susceptibility whose maximum and intermediate axes lie in a plane, named by Ising (1942)

the magnetic stratum plane. In these measurements, this plane is never inclined at more than 30° to the horizontal. The direction of maximum susceptibility dips downwards at an angle of 8° to 25° in a direction between North and the direction of origin of the flow, while the intermediate axis is usually nearly horizontal. The magnitudes of the susceptibilities in the three directions are usually in the approximate ratios

$$k_a : k_b : k_c : = 107 : 100 : 87.$$

Table 2 gives the azimuth and inclination of the maximum and intermediate susceptibility axes. Since the absolute value of susceptibility is not given by the measurements made for the determination of anisotropy, Table 2 gives instead the values of μ_a, μ_b and μ_c which are the differences of k_a, k_b and k_c respectively from their mean. These parameters together with the mean susceptibility, assumed here to be constant, are sufficient to define a triaxial ellipsoid.

Table 2

Results of the measurements of anisotropy of magnetic susceptibility

	Max. Susc.		Int. Susc.		Mean × 10⁶emu/cm³			Mean ρ_a
Θ_F	Mean D deg	Mean I deg	Mean D deg	Mean I deg	μ_a	μ_b	μ_c	
1·03	25	11	−65	0·5	3·2	1·1	−4·3	31
0·81	18	19·5	−72	1	3·6	0·5	−4·1	17
0·78	24	13	−66	−0·5	3·8	1·1	−4·9	29
0·65	19	18	−71	−2	3·6	0·5	−4·1	17
0·55	25	17·5	−65	−0·5	3·9	0·7	−4·6	31
0·49	18	19	−71	−2·5	3·6	0·5	−4·1	17
2·0	63	9	−28	4	3·0	1·8	−4·8	52
1·36	50	15·5	−39	1	3·3	1·7	−5·0	31
0·98	50	12	−41	4·5	—	—	—	31
0·61	36	21·5	−54	−0·5	3·2	0·8	−4·0	18
0·40	32	20	−58	0·5	3·3	0·7	−4·0	15·5
0·26	29·5	21	−61	0	3·3	0·7	−4·0	13
0·02	24	20	−65	0	3·2	0·6	−3·8	10·5
0·01	18	20	−72	−1·5	2·8	0·8	−3·6	7·5
0·0	9	19	−80	−1	2·8	0·6	−3·4	3·5
1·18	133	10	42	8·5	2·3	1·0	−3·3	—
0·81	124	15	31	7·5	2·3	1·1	−3·4	56
0·65	117	14	24	9	2·2	1·1	−3·3	44
0·30	69	26	−23	2·5	1·8	0·9	−2·7	22
0·20	57	26	−33	1	1·6	1·1	−2·7	19·5
0·12	37	24·5	−50	−7	1·7	1·0	−2·7	8

Figure 4 is a stereogram showing the directions, in the lower hemisphere, of the axes of maximum susceptibility of all the specimens measured. It will be seen that these directions can be divided into three groups corresponding to the directions of flow in which the sediments were deposited, and that each group lies nearly on a great circle which passes through the horizontal at the direction of origin of the flow and through a direction dipping 20° downward in the meridian.

As will be shown in a later section, it has been found possible to explain the directions of the axes of maximum susceptibility by a model similar to the "rolling ball" model of the remanence theory. This leads to the prediction of effects analogous to the inclination error δ and the rotation ρ which will be denoted by the

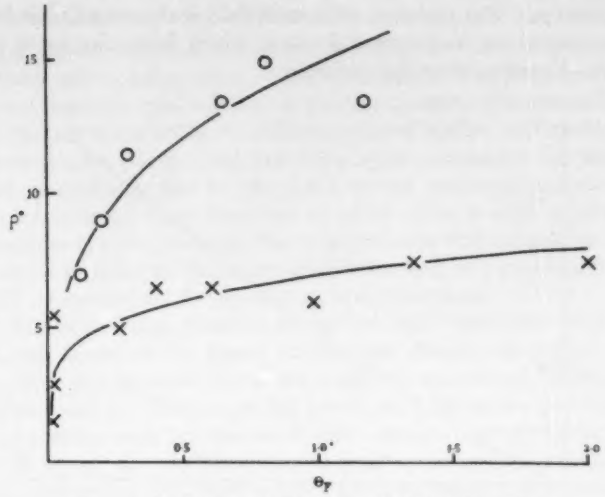

FIG. 3. Variation of rotation ρ with fluid stress Θ_F.
O—flow from 145° ×—flow from 90°.

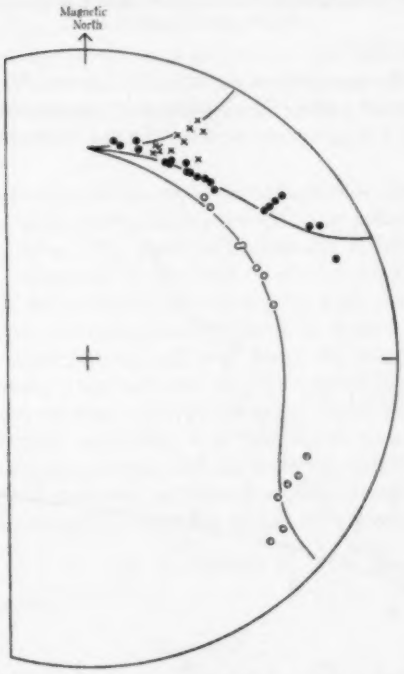

FIG. 4. Directions, in the lower hemisphere, of the axes of maximum susceptibility of all specimens measured—stereographic projection. The full lines show the expected paths of movement of the maximum susceptibility direction with increasing stress
×—Flow from 45° ●—Flow from 90° O—Flow from 145°

symbols δ_a and ρ_a. The variation of δ_a with field inclination I_F has been studied in the two experiments in the small flume in which fields dipping at 40° and 20° were applied. Figure 5 shows this variation.

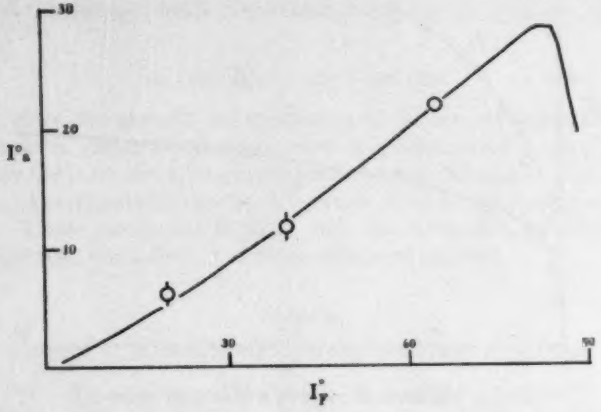

FIG. 5. Variation of the inclination I_a of the maximum susceptibility direction with field inclination I_F. The full line is a theoretical one derived from equation 15.

Figure 6 shows the variation of ρ_a with Θ_F for two flow directions. As with the remanence rotation ρ there is a variation in magnitude with the direction of flow, the largest values of ρ_a being found for the 145° azimuth.

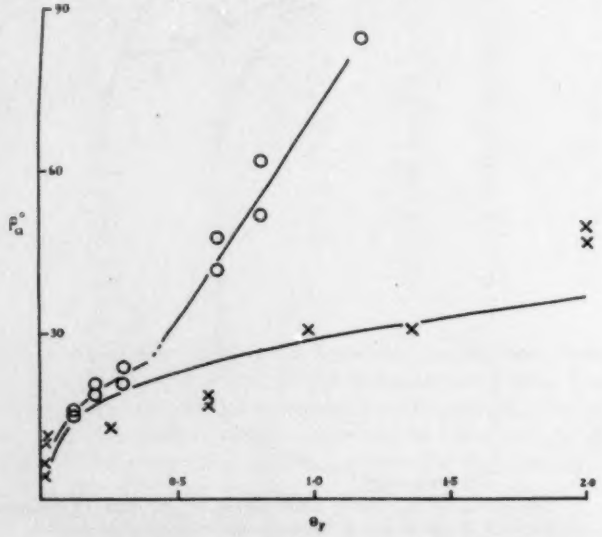

FIG. 6. Variation of the rotation ρ_a with fluid stress Θ_F.
O—flow from 145°
×—flow from 90°

5. An analysis of the experimental results

In Section 1 an outline has been given of the theory advanced to explain the inclination error and bedding error of the remanence. The postulated rolling of quasi-spherical particles into hollows in the bed accounts quantitatively for these effects, and may be extended in a qualitative way to explain the rotation effect.

It has been shown (Rees 1959) that for a typical sediment, the anisotropy of susceptibility is probably due to the shape of the magnetic particles, and any theory of the anisotropy must bear this in mind. The average sphericity of the magnetic particles is such, however, that it is probable that they behave mechanically very much like spheres. Assuming such behaviour it is possible to extend the rolling theory to account for the anisotropy of the sediment.

That it has been found possible to explain both remanence and anisotropy effects as being caused in the above manner has obvious advantages, since it is possible to relate the remanence and anisotropy measurements made on an individual specimen and to obtain from the combined information predictions which would not be possible were the two sets of data unrelated. In particular, it has been found possible to use the combined data to find, under certain circumstances, the direction and strength of flow from which a given sediment was deposited, and the azimuth and dip of the magnetic field present during deposition. This problem involves four unknowns, and clearly cannot be solved from measurements of direction of remanence or maximum susceptibility alone as each provides only two items of information.

The combination of remanence and anisotropy data will be discussed later in this section, but first the new remanence results will be examined in the light of the theory given previously, and an account given of the new theory of the anisotropy.

(a) *The rotation effect*

The rotation ρ which takes place when the sediment is deposited from flowing water may be thought of as being due to the shearing action of the fluid in the laminar sub-boundary layer. The effect will be similar in form to that due to the shearing effect of the tangential component of gravity when the sediment is deposited on a slope. If this comparison is accepted then the shearing forces are usually, within the limits observed, about the same as those to be found on a slope of from 5° to 10° or between 0·09 and 0·18 times the weight of the particles.

If the boundary between the sediment and the depositing fluid were perfectly plane it would be true to say that, over the boundary layer, the tangential stress $\mathcal{T}$ was constant. Under these conditions it is possible to consider the equilibrium between the magnetic aligning couple and the shearing couple of the fluid.

Considering a layer of magnetic particles in a plane parallel to the bed sheared through an angle ρ, the magnetic restoring couple on a particle of volume V is

$$C_m = VIH \sin \rho \tag{8}$$

and the shearing moment

$$c_s = \mathcal{T} V. \tag{9}$$

For equilibrium

$$\mathcal{T} = IH \sin \rho \tag{10}$$

$$\frac{\mathcal{T}}{\gamma} = \Theta = \frac{IH \sin \rho}{(d_g - d_f) g D \cos \beta} \tag{11}$$

or for magnetite in water approximately

$$\Theta = \frac{1}{3}\frac{IH\sin\rho}{gD}. \tag{12}$$

Now the basic assumption of the rolling theory is that particles roll into hollows in the bed on touching down and that they are not inhibited from doing so by the presence of the magnetic aligning forces or, in terms of the magnetic and gravitational couples.

$$VIH < DVgd_g \tag{13}$$

or, from (12) and (13)

$$\Theta < \tfrac{1}{3}\sin\rho. \tag{14}$$

Since the rotation ρ which has been observed is of the order of 10° for values of the fluid stress as high as $\Theta_F = 2{\cdot}0$, the stress Θ acting on the particles near the bed must be very much less than the fluid stress.

No exact explanation can be given, as yet, for the form of the variation of ρ with Θ_F. Although it is clear that the effective stress at the bed is not linearly related to Θ_F, it is not yet possible to predict its value from Θ_F. It has been shown (Bagnold 1956) that the formation of bed features, and the sediment load in a stream both have a marked effect on the value of the effective stress found for a given value of fluid stress, but there is not, as yet, direct evidence of the magnitude of these effects for very fine sediments.

It has been found in these experiments that the magnitude of ρ for any given value of Θ_F varies according to the direction of flow, and it is difficult to see how any theory which considers the particles as spheres can account for this. If, however, remanence direction is to some extent related to particle shape then the possibility arises that the shearing forces acting on the aligned particles are more effective when acting in some directions than they are in others, the difference being due, perhaps, to the variation in hydrodynamic lift with the attitude of the particles to the stream direction. Particles with a negative angle of attack—i.e. dipping down into the stream direction, have been shown experimentally to be more stable on the bed than those with a positive angle of attack (Rusnak 1957). This suggestion receives additional support from the anisotropy data as will be shown later.

If remanence direction is related to shape then the original theory of King (1955) must to some extent be applicable. Since both this theory and the present theory predict identical results it is not possible to separate directly their contributions to the inclination error. It has been found, however, that a very plausible value of the average angle of roll will, according to the present theory, account quantitatively for the inclination and bedding errors, and, furthermore, the shape analysis suggests that the contribution to the inclination error due to the effect of remanence controlled by particle shape is small.

(*b*) *An extension of the rolling theory to account for the anisotropy of susceptibility*

The rolling theory has been extended to account for the anisotropy results. Although the anisotropy is thought to be due to the alignment of the long axes of non-spherical particles the shape analysis and the success of the rolling theory for the remanence effects suggest that most of the particles behave, from a mechanical point of view, not very differently from spheres. The model adopted is of 4N

particles of magnetically soft material which approximate mechanically to spheres and magnetically to prolate ellipsoids with principal susceptibilities k_1, k_2 and k_2 ($k_1 > k_2$). The induced magnetization in the particles will tend to be along their directions of maximum susceptibility, and it is assumed that in still water this will cause a perfect alignment of the k_1 direction in the field. The model assumes further that when the particles touch bottom, N roll an average angle ϕ about each of four horizontal axes in the north, south, east and west directions.

If the anisotropies after rolling are added as tensors the inclination of the resultant direction of maximum susceptibility can be found. This is given by the expression:

$$\tan 2I_a = \frac{\sin 2I_F(\cos\phi + \cos 2\phi)}{2\cos 2I_F \cos^2\phi + \sin^2 I_F \sin^2\phi} \qquad (15)$$

where I_F is the field inclination and I_a the inclination of the resultant maximum susceptibility. The declination of the maximum susceptibility is 0°.

The variation of I_a with ϕ for a value of I_F of 65° is shown plotted in Figure 7. In one experiment the water was moving very slowly and this may be regarded as a still water experiment. The value of I_a for this experiment was 22° giving

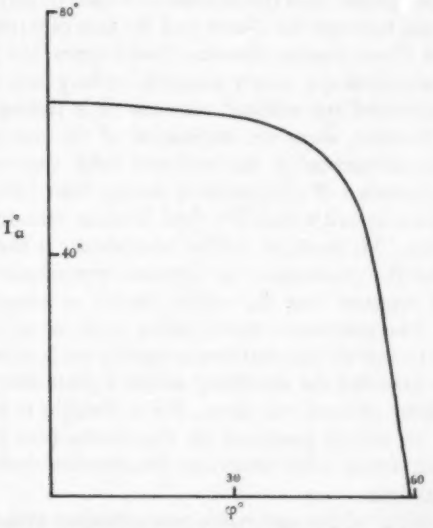

FIG. 7. Variation of the inclination I_a of the maximum susceptibility direction with the rolling angle ϕ for a field inclination of 65°.

from Figure 7, a value of ϕ of 56°. Using this value of ϕ the variation of I_a with I_F has been plotted (Figure 5) together with the mean values of I_a in the two experiments which were carried out at lower field inclinations to test this theory. The agreement is sufficient to suggest that the theory is a fair approximation. The value of ϕ of 56° is less than that found for the remanence rolling. This may be due to the departure from perfect sphericity of the particles responsible for the anisotropy being greater than that of those responsible for the remanence, their rolling thus being more restricted.

If the rolling theory can be applied to the anisotropic particles it would be expected that besides an effect analogous to the inclination error there would be others like the bedding error effect on sloping surfaces and the rotation effect in moving water. The former has not yet been investigated, but it has been found that the results obtained from flowing water can be explained in terms of this model. The full lines on the stereogram showing the directions of maximum susceptibility (Figure 4) have been plotted as follows: A small circle has been drawn for each azimuth of flow corresponding to the expected rotation path—i.e. with its axis horizontal and at right angles to the direction of flow—and from this the expected path of the axis of maximum susceptibility has been constructed taking into account the variation of I_a with I_F. These curves are seen to fit the experimental results and provide more support for the view that the model is a reasonable approximation.

The most obvious alternative explanation of the anisotropy results is in terms of a sediment composed of plate-like or elongated particles which are oriented in the bed by hydrodynamic and gravity forces, rolling playing only a secondary part because of the strong preferred orientation of such particles on a stream bed.

If the sediment is supposed to contain some plate-like and some elongated particles then the plates will tend to align in a plane near the horizontal to give the "magnetic stratum plane" and the needles will tend to give a maximum in this plane lying somewhere between the North and the axis of current flow.

The theory given above implies that this does happen, but it might be thought possible that the particle shape might preclude rolling and that the anisotropy results could be accounted for without recourse to a rolling theory. It seems, however, to be fairly clear, since the inclination of the maximum susceptibility axis varies with the inclination of the ambient field, that magnetic forces are important in the orientation of the particles during their fall. If this is so then rolling must take place in order that the final laminar structure of the sediment can be brought about. To preclude rolling completely is therefore not possible on this evidence, and the quantitative fit between experiment and theory shown in Figures 4 and 5 suggests that the rolling theory is adequate to explain the experimental facts. The postulated mean rolling angle of 56° is less than that of 70° found necessary to explain the remanence results yet is more than the value of 45° which would be expected for randomly oriented plate-like particles falling on the bed and, as has been pointed out above, this is thought to be accounted for by a partial restriction on rolling produced by the flattened or elongated character of the particles which makes their behaviour intermediate between that of plate-like and spherical particles.

The rotation angle ρ_a, of the maximum susceptibility axis varies with applied stress Θ_F and, like the angle ρ, with the direction of flow. Assuming that the variation of ρ_a with Θ_F is of the form $\rho_a = A\,\Theta_F^n$ a least squares fit of $\log \rho_a / \log \Theta_F$ gives

$$\rho_a = 29.2\,\Theta_F^{0.343 \pm 0.037} \tag{16}$$

for flow from 90° and

$$\rho_a = 64.1\,\Theta_F^{0.728} \tag{17}$$

for flow from 145°.

The variation of $\log \rho_a$ with $\log \Theta_F$ for flow from 145° shows a distinct break at a value of Θ_F of about 0·5, the parts of the graph above and below this value

having significantly different slopes. Below $\Theta_F = 0{\cdot}5$ the variation is best fitted by the equation

$$\rho_a = 37{\cdot}2\ \Theta_F^{0{\cdot}408\pm0{\cdot}067} \tag{18}$$

and above by

$$\rho_a = 71{\cdot}1\ \Theta_F^{10{\cdot}69\pm0{\cdot}144}. \tag{19}$$

Such a break in slope may also be present in the graph of $\log \rho_a / \log \Theta_F$ for flow from 90° though this is not proved.

The variation of ρ_a with Θ_F for flow from 90° over the whole range of Θ_F and that of ρ_a with Θ_F for $\Theta_F < 0{\cdot}5$ for flow from 145° has been compared with the variation of ρ with Θ_F. The values of n in the expression $\rho = A\,\Theta_F^n$ for the remanence measurements do not, for values of Θ_F less than 0·5, differ significantly from those for the anisotropy measurements. This suggests that both may obey the same law for values of stress below $\Theta_F = 0{\cdot}5$, that is for relatively gentle flow and so an attempt has been made to find if there is a similar variation of ρ with ρ_a within the range $0 < \Theta_F < 0{\cdot}5$ (Figure 8). It has been found that there is a very good correlation between ρ and ρ_a, the best line being

$$\rho = -1{\cdot}30 + (0{\cdot}568 \pm 0{\cdot}052)\rho_a. \tag{20}$$

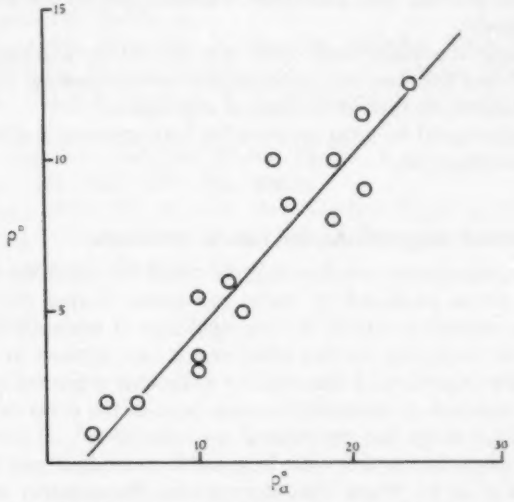

FIG. 8. Variation of the rotation ρ with the rotation ρ_a for values of Θ_F less than 0·5.

This gives a method of estimating ρ directly from ρ_a for low values of Θ_F which, since it avoids the intermediate step of estimating Θ_F is likely to be of particular use in the correction of natural remanence directions.

6. Application to the estimation of magnetic directions

The principal object of this work has been to find a way of correcting the direction of magnetization of natural sediments for the effects of water currents. This requires an estimate of ρ and an estimate of the direction of water movement.

The latter may sometimes be obtained from geological evidence, though in the relatively quiet water from which the Swedish varves were deposited, such evidence will probably be rare.

The value of ρ may be estimated by measuring some parameter which is known to be related to it, the most obvious being the fluid stress Θ_F and the parameter ρ_a found from the anisotropy measurements.

At low values of the fluid stress the best method of estimating ρ appears to be from the value of ρ_a, using equation (20), the correlation being extremely close. This method has the advantage that it avoids the necessity of estimating Θ_F, an operation which presents considerable problems even in a laboratory investigation.

For values of Θ_F less than 0·5 it has been shown that there is a very close correlation between ρ and ρ_a, the value ρ/ρ_a being 0·57 $\pm$ 0·05 for the silt used in the present series of experiments. This is true for both the azimuths of flow which have been studied. Now both ρ_a and ρ are defined as rotations, along the same small circle, which take place before rolling and both those rotations are defined as starting from the field direction. If the directions of remanence and maximum susceptibility are known and corrections for δ and δ_a are made, then the field direction must lie on a small circle whose axis is horizontal and which passes through the two corrected directions. Furthermore the ratio of the angles between the field and the two directions is known and so the field direction is determined uniquely.

Other estimates of applied fluid stress may be obtained, it appears, from the measurement of bed features, or, in favourable circumstances, from the known slope of the surface of the flow at the time of deposition.

These methods would be most accurate for high stresses, and are not relevant to the present investigation.

7. Conclusions and suggestions for future research

In this paper experiments are described in which the effect has been measured of the shearing stress produced by water movement during deposition on the directions of the remanence and of the principal axes of susceptibility. A method has been found of correcting for this effect and it now appears to be possible to correct back to the original field direction for sediments deposited on a flat bed.

The secular variation in modern times has been of the order of 1° in 10 years with a total variation in the last 400 years of the order of 10°. If useful data on the secular variation are to be found it must be possible to estimate past field directions to an accuracy of 2° or 3°. From laboratory results, the accuracy of estimation of the field direction from a single specimen given the directions of remanence and maximum susceptibility may well be as low as 2° or 3°, so, if these results are applicable to nature, it should now be possible to find the secular variation from measurements on flat lying varves.

It is, in principle, not difficult to test the applicability of these results in the field by a method similar to that used to test the bedding error results (Griffiths 1955, Griffiths & others 1960). Two or more sites are chosen in which the varves are of the same age and the magnetization directions are compared before and after correction for inclination error and rotation. If the method is applicable there will be a significant decrease in the mean difference between magnetization directions from the different sites, and, if all disturbing factors have been allowed for, the mean difference should be of the order of 3° or less.

Acknowledgment

I wish to thank Dr D. H. Griffiths and Dr R. F. King for their help. This work has been intended to supplement theirs, and I have had the benefit of their experience throughout, many of the ideas, otherwise unacknowledged here, being originally due to them.

The collection of the material from Sweden was made possible by grants to Dr Griffiths from the Royal Society and from the University of Birmingham who also made a grant towards the cost of the apparatus. My thanks are also due to Professor F. W. Shotton, F.R.S., for the use of the facilities of this Department and to Mr C. S. Bright who constructed the apparatus.

Department of Geology,
University of Birmingham:
1961 *January.*

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Research Notes

A Note on the Seismicity of the Ross Sea Region

T. Hatherton

(Received 1960 December 22)*

Summary

Although the aseismicity of Antarctica has been confirmed by IGY observations, small local disturbances, up to magnitude 3, are almost a daily feature of seismograms at Scott Base. *S–P* intervals of these shocks range from 3 to 38 s but half of the shocks recorded have *S–P* differences of 4–6 s, values appropriate to the distance of Mt Erebus. An unusual low-velocity phase ($V \triangleq 0{\cdot}6$ km/s) is present on many seismograms.

For many years it has been known, from teleseismic observation, that the whole of Antarctica is seismically stable. No epicentre has yet been reliably located along the postulated structural connection between the junction of the Easter Island Rise, the Indian Antarctic Swell and the New Zealand region at about 64° S, 160° E, and the northern end of the Palmer Peninsula; or at latitudes higher than 65° S elsewhere (Gutenberg & Richter 1949). As a result of the comparatively close network of seismological stations on the Antarctic continent developed during IGY, minor seismicity, previously undetected teleseismically, should be disclosed. Activity of sufficient magnitude ($\sim M = 4$) to be located by the present array of stations is also absent in the Ross Sea region. Examination of the Scott Base and Hallett Station records during IGY reveals no shocks which could be placed south of the Balleny Islands, with the exception of minor local activity near Scott Base.

This activity, being recorded at only a single station, does not lend itself to reliable quantitative analysis. However the following remarks may be made following a study of these local shocks.

1. Over 300 near earthquakes were recorded at Scott Base between 1957 June 1 and 1958 December 31. Provisional *S–P* intervals were assigned to 150 of these shocks and these intervals range from 3 to 38 seconds. Seventy-six earthquakes have *S–P* intervals of 4–6 s; the remainder are mainly grouped in the 9–13 s and 27–38 s ranges, the latter values being dubious because of the difficulty in identifying the first *S* arrival. In connection with the origin of shocks of *S–P* interval 4–6 s it is worth noting that the Scott Base seismographs are about 30 km south of the mildly active volcano Mt Erebus.

* Received in original form 1960 November 11.

2. The maximum magnitude of the local earthquakes recorded during the above period is estimated to be not greater than 3·0

3. On many of the seismograms an unusual phase of apparent velocity about 0·6 km/s is present. Most prominent on the vertical component, this arrival takes the form of one or two pulses of 4–8 cycles of regular waves of period about 1½–2 s. Figure 1 shows the vertical component of a typical earthquake in the

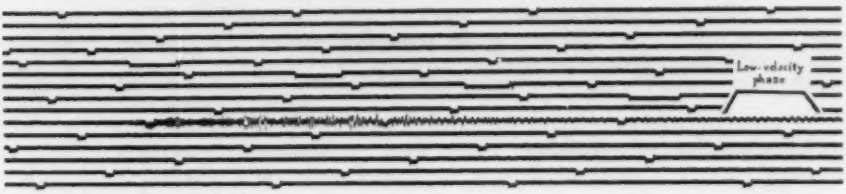

FIG. 1.—Local earthquake recorded on short-period vertical seismogram (Benioff), 1957 August 3, at Scott Base.

9–13 s range and the diminishing coda increases in amplitude and period and becomes quite regular as the low-velocity phase arrives. Kizawa (1960) has recently reported a phase with a group velocity of 0·6 km/s on records from the Mori weather station 54 km from the volcano Usu in Japan. Kizawa's "third phase" shows dispersion, stationary values of the group velocity lying between 1½ and 2 s. The path between Usu and the Mori weather station passes through Utiura Bay (depth < 100 m) for over 90 per cent of its length and the phase is not recorded at other stations in the vicinity where there is no water covered segment in the path.

Geophysics Division, D.S.I.R.
Wellington, New Zealand:
1960 *November.*

California Institute of Technology,
Division of the Geological Sciences
Contribution, No. 991.

References

Gutenberg, B. & Richter, C. F., 1949. *The Seismicity of the Earth*, Princeton University Press.

Kizawa, T., 1960. *Geophys. Mag.*, **29**, 477–498.

Some 1951 Earthquake Mechanisms based on P and PKP Data

A. R. Ritsema

(Received 1961 January 27)

1. Presentation of solutions

The methods used here have been described in an earlier paper in this periodical (Ritsema 1960). The greater part of the data on which the solutions are based is taken from the International Seismological Summary. Additional data have been supplied by the respective seismic stations in reply to questionnaires. The co-operation of the Directors of these stations is gratefully acknowledged.

In Table 1 the data not reported in the I.S.S. are given. In Table 2 the solutions reached have been summarized. It is impossible to state which of the two nodal planes acted as fault plane because of the fact that only longitudinal waves have been used. Only one of the two possible solutions is given in the table. The other solution is found by interchanging the *A*- and *C*-axes and by changing the dextral (respectively sinistral) part of the fault motion into a sinistral (respectively dextral) part.

To give an insight into the reliability of the solutions the numbers of consistent and of inconsistent data are given in the last column of Table 2.

2. Remarks

In earthquakes nos. 3, 4, 11, 12, 13, 15, 16 and 20 a greater or smaller variation is possible in the position of one of the nodal planes. The two extremes are given in the table of solutions.

The high percentage of inconsistent data in earthquakes nos. 10, 14, 18, 19, 20 and 24 can be explained by one of the nodal lines passing near to a group of inconsistent stations. Peculiar inconsistencies, however, sometimes do occur, as for example those of Irkutsk and Alma Ata in earthquake no. 1; of Irkutsk, Tashkent and Poona in earthquake no. 6; of Cleveland, Columbia, Ottawa and Bermuda in earthquake no. 15; and of the *PKP* of Tashkent, Poona and Hyderabad in earthquake no. 24. Such groups of inconsistent stations amidst consistent data of the opposite kind are not readily explained.

In earthquake no. 7 either the West block is moving vertically downward with respect to an upward moving East block, or the upper block is moving towards the East with respect to a Westward moving lower block.

In earthquake no. 24 the onsets of the nearby Mexican stations can only be explained by a position of the focus inside the crust and not in the upper mantle as suggested by the depth of the focus of 0·00 R reported in the I.S.S.

Note that in the case of earthquake no. 5 an epicentre has been assumed about 1°WNW of the location given in the I.S.S. This position seems to be in better accord with the travel-time data.

Table 1

Additional data not reported in ISS

Earthquake No. (see Table 2)	Data
1	Alma Ata (*C*); Irkutsk (*C*); Poona *D*.
2	Hungry Horse *C*; La Paz (*D′*); Pasadena *C*; Tashkent *C*.
3	Irkutsk *D*; København (*D*?); La Paz *C′*; Scoresby Sund *C*.
4	Alma Ata *C*; Tashkent *C*; Warszawa *C*; Ottawa (*C*?).
5	Irkutsk *D*; Alma Ata *D*; Hyberabad (*D*); Tashkent *D*; College (*C*); Warszawa *D*.
6	Irkutsk (*C*); Alma Ata *D*; Tashkent (*C*); Poona (*C*); Warszawa (*C*); Witteveen *D*; La Paz *C′*?.
7	Clermont Ferrand (*D*?); Wien *D*; Praha *D*; Ksara *D*; Ottawa (*D*?).
8	Fukuoka *C*; Abuyama *D*; Alma Ata *D*; Poona *C*; Tashkent *D*; Perth *C*; College (*C*); Warszawa *D*; Scoresby Sund *D*; Reykjavik *D*; Hungry Horse *D*; Pasadena *C*.
9	Irkutsk *C*; Alma Ata *C*; Tashkent *C*; Poona *D*; Bombay *D*; Uppsala *C*; Ksara *C*; Warszawa (*D*); Wellington *C*?; Praha *C*; Reykjavik *D*; La Paz *D′*.
10	Bergen (*D*); Uppsala *C*; København *C*; Warszawa *C*; Athena *C*; Ottawa *D*; College (*D*); Ksara *D*; Tashkent *D*?; Irkutsk (*C*); Alma Ata *D*; Tamanrasset (*D*); Florissant *D*?; San Juan *C*?; Poona (*C*?).
11	Columbia *D*; Florissant *D*; Bogota *D*; Cleveland *D*; Palisades *D*; Ottawa *D*; Hungry Horse *C*; College *C*; Scoresby Sund (*C*?); Toledo *D*.
12	College *C*; Scoresby Sund *C*?; Hungry Horse *C*; La Paz *D′*.
13	Irkutsk *C*; College *C*; Alma Ata *C*; Tashkent *C*; Hungry Horse *C*; Praha *C*; Palisades *C*.
14	Trieste (*C*); Wien *D*; Neuchatel *D*; Clermont Ferrand *D*?; Warszawa *C*; København (*C*?); Toledo (*D*); Ksara *C*; Tashkent *D*?; Alma Ata *D*.
15	Honolulu *C*; College *C*; Abuyama *D*; Columbia (*D*?); Ottawa (*D*); Bogota *C*; Bermuda (*D*?) Scoresby Sund *C*; Alma Ata *C*; Warszawa *C*; Ksara (*D′*?).
16	Irkutsk *C*; Alma Ata *C*; Tashkent *C*; Poona (*D*); Warszawa (*D*); Praha *C*; Helwan *C*.
17	Irkutsk *C*; Alma Ata *C*; Tashkent *C*?; Poona *C*; Kiruna *D*?; Witteveen *D*; La Paz *D′*?.
18	Irkutsk *C*; College *C*; Alma Ata *C*; Tashkent *C*; Hyderabad *C*; Poona (*D*); Scoresby Sund (*D*); Hungry Horse *C*; Uppsala *C*; Warszawa (*D*); de Bilt *C*; Ottawa *C*; Palisades (*D*).
19	Irkutsk *C*; College *C*; Alma Ata *C*; Tashkent *C*; Poona (*D*); Hungry Horse *C*; La Paz *C′*?.
20	Irkutsk *C*; College *C*; Alma Ata *C*; Hungry Horse *C*; Tashkent *C*; Scoresby Sund *C*; Hyderabad *C*; Reykjavik *C*; Poona (*D*); Warszawa *C*; Ksara *C*; Palisades *C*; Perth *C*; Christchurch *C*; Alger (*D*); La Paz (*D′*).
21	Tananarive *C*; Poona *D*; Ksara *D*; Riverview *C*; Tashkent *D*; Alma Ata (*C*); Christchurch (*D*); Warszawa *D*; Irkutsk *D*; Kiruna (*C*?); Bogota *C′*; Honolulu *D′*.
22	Tucson *D*; Hungry Horse *C*; Tacubaya *D*; College *C*; San Juan (*D*); Bogota *C*; La Paz *C*; Witteveen *C*; Toledo (*D*); Poona *C′*.
23	Irkutsk *D*; Tashkent *C*; Hongkong *C*; Hyderabad *D*; Poona *D*; Warszawa *C*; Praha *C*?; College (*C*); Hungry Horse *C*.
24	Tucson *C*; Bogota *C*; La Paz *C*; College *C*; La Plata *C*; Scoresby Sund *C*; Toledo *C*; Witteveen *C*?; Praha *C*?; Warszawa (*D*); Wellington *D*?; Christchurch (*C*); Tashkent (*D′*); Poona (*D′*); Hyderabad (*D′*); Tananarive *C′*.

Remarks: *C*: compressional *P* wave
D: dilatational *P* wave
C′: compressional *PKP* wave
D′: dilatational *PKP* wave
( ): inconsistent with the solution reached.

Table 2

The solutions reached

No.	Date 1951	Time h m s	Epicentre Lat. deg.	Long. deg.	Depth	A-axis (or C-axis) Azimuth	plunge	B-axis Azimuth	plunge	C-axis (or A-axis) Azimuth	plunge	Type	Number of data consistent	inconsistent
1	Jan. 8	18 32 28	35°·6 N	140°·0 E	0·005 R	N 315° E	28°	N 45° E	0°	N 135° E	62°	T	39	6
2	Feb. 2	21 02 56	37°·1 N	141°·8 E	0·00 R	N 144 E	55	N 29 E	16	N 289 E	30	PL	19	3
3	Feb. 7	03 38 48	31°·2 N	140°·4 E	0·01 R	N 228 E	4	from N 115 E to N 138 E	79 4	from N 319 E to N 360 E	10 84	RT TR	31	2
4	Feb. 10	08 38 14	43°·9 N	146°·2 E	0·01 R	N 90 E	6	from N 353 E to N 360 E	50 0	from N 185 E to N 270 E	40 84	RT T	20	1
5	Mar. 5	20 11 48	28°·2 N	128°·7 E	0·03 R	N 295 E	30	N 133 E	59	N 30 E	8	RT	51	7
6	May 4	11 53 13	44°·4 N	142°·2 E	0·03 R	N 264 E	5	N 170 E	45	N 359 E	45	RT	39	5
7	May 15	22 54 26	45°·5 N	9°·6 E	shallow	N 105 E	0	N 15 E	0	—	90		20	3
8	May 31	20 56 01	18°·6 N	121°·2 E	0·01 R	N 176 E	11	N 70 E	54	N 273 E	33	LP	28	5
9	June 5	16 57 45	29°·8 N	131°·2 E	0·005 R	N 93 E	8	N 281 E	82	N 183 E	1	LP	42	8
10	June 6	16 10 48	71°·3 N	9°·7 W	shallow	N 118 E	22	N 278 E	66	N 25 E	7	RT	30	8
11	July 9	00 03 47	16°·0 N	96°·5 W	shallow	N 229 E	18	from N 121 E to N 139 E	44 0	from N 335 E to N 49 E	40 72	LP P	21	4
12	July 9	01 31 03	33°·1 N	139°·3 E	0·025 R	N 210 E	10	from N 320 E to N 303 E	63 17	from N 115 E to N 90 E	25 70	LT TL	33	0
13	July 26	09 59 59	40°·9 N	142°·7 E	0·005 R	N 113 E	45	from N 356 E to N 14 E	24 9	from N 248 E to N 275 E	35 43	PR PR	26	3
14	Aug. 8	20 56 31	42°·6 N	13°·5 E	shallow	N 282 E	34	N 148 E	46	N 30 E	24	LP	17	4

15	Aug.	21	10	56	58	19°·2 N	155°·5 W	shallow	N 38 E	0	from N 128 E to N 128 E	60 0	from N 308 E to —	30 90	R	25	5
16	Aug.	24	14	21	35	46°·5 N	150°·7 E	0·025 R	N 320 E	9	from N 60 E to N 53 E	48 17	from N 222 E to N 202 E	40 81	RP PR	31	4
17	Oct.	15	21	01	57	32°·6 N	134°·5 E	shallow	N 211 E	18	N 331 E	58	N 112 E	36	LT	28	5
18	Oct.	18	08	26	24	41°·6 N	142°·0 E	0·00 R	N 124 E	42	N 30 E	5	N 293 E	48	PL	60	15
19	Oct.	19	14	51	31	41°·6 N	142°·0 E	0·005 R	N 135 E	48	N 12 E	26	N 265 E	30	PL	25	6
20	Nov.	6	16	40	06	47°·5 N	153°·9 E	shallow	0	43	from N 250 E to N 110 E	20 20	from N 143 E to N 217 E	40 40	PL PR	35	13
21	Dec.	8	04	14	17	33°·7 S	57°·7 E	0·00 R	N 60 E	6	N 202 E	82	N 330 E	5	LT	32	5
22	Dec.	26	00	46	52	32°·8 N	118°·4 W	shallow	N 70 E	10	N 176 E	58	N 334 E	30	RP	21	3
23	Dec.	26	16	30	47	39°·2 N	95°·8 E	shallow	0	15	N 150 E	73	N 268 E	8	RP	24	3
24	Dec.	28	09	20	26	16°·9 N	98°·7 W	0·00 R	N 43 E	10	N 133 E	0	N 223 E	80	P	33	9

Remarks: A-axis—fault motion direction
B-axis—line of intersection of fault plane and auxiliary plane
C-axis—normal to fault plane
R—dextral part of the fault motion
L—sinistral part of the fault motion
P—reverse part of the fault motion
T—normal part of the fault motion

3. Conclusions

The conclusions of the previous paper are confirmed, namely a predominance of transcurrent fault motions over normal- and reverse-, and a preference of *A*- and *C*-axes for directions normal to or parallel with local trends.

Royal Netherlands Meteorological Institute,
de Bilt, Holland:
1961 *January* 27.

Reference

Ritsema, A. R., 1960. Focal mechanisms of some earthquakes of the year 1950. *Geophys. J.*, 3, 307.

Conference on Palaeomagnetism

King's College, Newcastle-upon-Tyne, 1961 April 20—22

This conference was organized by Professor S. K. Runcorn with the support of the Institute of Physics and the Physical Society, and was attended by over 60 people, most of whom are actively engaged in research in this field. It was apparent from the balance of subject matter in the papers presented that interest is divided about equally between the problem of the nature of the remanent magnetization of rocks and the broader geological implications of the large number of field studies of remanence that have now been made.

No new methods of measuring magnetic properties of rocks have been devised recently, and the papers on instruments were devoted to comparisons of methods in common use (*K. M. Creer, S. K. Runcorn*) with particular reference to the effect of specimen shape and inhomogeneity on measurements of weak intensities of remanence; a description by *L. Molyneux* of a computer programme for the reduction and statistical treatment of results; and an account by *J. A. As* of the astatic magnetometer at the Meteorological Institue, De Bilt, which has been used by the Dutch group of workers to produce a considerable body of data on European rocks. Instrumentation problems in the field of susceptibility anisotropy measurement were discussed by *C. G. A. Harrison* and *H. Noltimier*.

Discussion of the magnetization of sediments, in particular those red sediments whose remanence has been attributed to a thin haematite cement on the grains, occupied some three hours of the meetings, which lasted for less than sixteen hours in all. The papers ranged from the fundamental studies, bordering on the field of solid state physics, of *K. M. Creer* and *C. B. Like* on the production and unusual magnetic properties of fine-grained haematite, through those of *S. K. Banerjee*, who showed that in some cases the redness of a rock is dependent on its grain-size rather than its degree of oxidation, to the detailed analysis by *D. W. Collinson* of the black and red magnetic minerals separated from a Triassic sandstone. Both these minerals, of which the red predominates in quantity, appear to be nearly pure haematite, but there is an indication that the remanent magnetization may be associated with the black component. Histograms of intensity of remanence, plotted logarithmically, were shown by *S. K. Runcorn* to give an approximately normal distribution in many red rocks, and he urged that more intensity measurements should be analysed in this way with a view to possible estimation of the order of magnitude of the past geomagnetic field.

Experimental work on the detrital magnetization of sediments was described by *D. H. Griffiths* and *A. I. Rees*. They had measured both remanence and anisotropy of silts deposited in flumes and believed that such measurements could lead to determinations of current directions in suitable sediments as well as to corrections for the deflection of the remanence by the current. Sediments in which the magnetization may well be detrital are found in the deep oceans, and *M. J. Keen* described results he had obtained from cores taken from regions of both pelagic and turbidity current deposition in the North Atlantic.

The stability of natural remanence under uniaxial and hydrostatic pressure is naturally of considerable importance, and palaeomagnetic work has in the past been subjected to criticism on the grounds that pressure effects during and after the formation of a rock might invalidate the assumption that the remanence indicates the direction of the geomagnetic field at the time of formation. The experimental work presented bearing on pressure effects was rather reassuring, for although *S. Kume* found a considerable pressure effect on the acquisition of a chemical remanence by haematite (the remanence tending to lie perpendicular to the uniaxial pressure rather than parallel to the magnetic field), the effect became negligible if the goethite from which the haematite was formed was first mixed with quartz sand, presumably giving a better simulation of natural conditions. Similarly the work of *A. E. Musset* on the effect of elastic stresses showed that only the "softer" components of the remanence were affected, though these were altered irreversibly.

Field studies of the effect of stress were reported by *R. W. Girdler* and *S. Kume*, by *M. D. Fuller*, and by *M. A. Khan* and *R. F. King*, the two last papers being concerned mainly with the relationship of susceptibility anisotropy to the fabric of deformed rocks.

Six papers on the results of magnetically "cleaning" rocks (by heating them or subjecting them to the action of alternating fields in the absence of a steady field) were read. Two of these (by *R. Wilson* and *E. Thellier*) emphasized the value of thermal demagnetization in removing the remanence due to a secondary component of low Curie Point, and in estimating the temperature at which a rock has acquired a secondary partial thermoremanence due to reheating: two others (*F. H. Chamalaun* and *J. D. A. Zijderveld*) described rocks in which the *secondary* magnetization was of high coercivity and could therefore not be removed by A.C. demagnetization. *E. Irving*, who presented at the conference some of the more recent results of the palaeomagnetic group at Canberra, discussed the criteria by which one might judge when a collection of specimens had been subjected to the optimum degree of "cleaning".

In a short session on applications of rock magnetism, *J. C. Belshé* reported the progress of his investigation of the secular variation over the past few thousand years by measurement of archaeologically dated specimens, and *J. D. Cornwell* gave the results of a study of the remanence of some igneous rocks bordering the Dartmoor granite.

In the session on Polar Wandering and Continental Drift, results of *E. Irving*, *S. K. Runcorn* and *A. E. M. Nairn* confirmed the existing picture of polar movement as seen from Australia, North America and Africa, and *D. E. T. Bidgood* presented a polar curve for East Greenland which could be reconciled with earlier European results by a fairly small relative movement. The results of the Dutch group, reported by *J. Veldkamp*, could also be brought into agreement with each other and with those of other workers by the assumption of relative rotations of parts of Europe during the Alpine orogeny. However, a paper by *A. Roche* (read by *Prof. Thellier*) quoted results from Europe giving pole positions for the late Tertiary and early Quaternary much farther from the present poles than is often supposed likely, and the work of *V. Bucha* on Czechoslovak rocks leads to a polar curve in which the change in longitude of the pole during Lower Palaeozoic time is eastward rather than westward as in curves which have hitherto been published. This disagreement added some weight to the written contribution from the U.S.S.R. by *Professor Beloussov*, who expressed the opinion that all palaeomagnetic results so far obtained can be accounted for by an extremely rapid polar wandering

without any relative movement of the continents. *Professor Runcorn*, however, maintained that this was not so.

The final session was devoted to the comparison of palaeoclimatic with palaeomagnetic data. When this comparison is made on a broad scale, as in the paper by *P. M. S. Blackett*, the correspondence between the conclusions drawn from very different fields of study is indeed remarkable, but later speakers (*A. E. M. Nairn* and *D. J. Blundell*) tended rather to emphasize the difficulties in making more detailed comparisons of this kind, due to the inherent uncertainties in both fields and the difficulty of time correlation between them. A paper by *W. B. Harland* used palaeomagnetic data to show that some apparently glacial Pre-Cambrian rocks in the Arctic were formed in low latitudes. He concluded that world-wide glaciation must have taken place at this time. An interesting feature of this session was the account given by *E. D. McKee* of the U.S. Geological Survey of the synthesis of the very large amount of geological information which is available in the U.S.A., largely as a result of drilling.

Such syntheses are unfortunately not available elsewhere, and until they are, (and perhaps even then, if local and rapid variations of climate are as common as some speakers supposed) it seems to be essential that palaeomagnetic results shall be compared with palaeoclimatic evidence only from precisely the same horizon.

One interesting idea was put forward by *Professor Runcorn*. This was that the splitting up of an original single continental mass might be explained by a combination of convection currents in the inside of the Earth with a change in modes of oscillation of these currents dependent on the size of a gradually increasing liquid core. There is no doubt that the large geophysical effort being placed in the direction of palaeomagnetic studies is bearing good fruit.

A Theory of Mountain Building

The problem of mountain building is a complicated one. This complexity is emphasized when we consider the rapidity of erosion. One way of finding an explanation of the facts is to consider tidal flexing of the Earth's crust combined with the assumption that the crust is not perfectly elastic and that the creep is faster in tension than it is in compression.

The postulates of this theory are:

1. The crust of the Earth is continually expanding horizontally.
2. This horizontal expansion is caused chiefly by the tidal flexing of the crust.
3. The pressures caused by horizontal expansion of the Earth's crust are relieved by folding, faulting and volcanism in zones of weakness which ultimately become zones of granitic accumulation.
4. The continual horizontal expansion of the Earth's crust during the history of the Earth helps account for the concentration of granitic materials of the crust into continental masses.

We have been led to postulate these ideas by some experiments on flexing of various materials in model studies over the past three years. We should be interested to hear of similar experiments or of any experiments which oppose or support the theory.

It is hoped that this letter will induce someone to measure the relative rates of creep in tension and compression for rock materials so that the theory can be tested mathematically.

Montana State College,
Bozeman,
Montana, U.S.A.
1961 *June* 13.

RAY WOODRIFF
MARJORY GOERING

Letters to the Editor

Isostatic anomalies and crustal structure in the Southern Cape

We regret to report that an error occurred in the above paper (Hales & Gough 1960. *Geophys. J.*, **3**, 225).

p. 231, line 7, should read:

"H being the thickness of the crust and ρ' = density of the mantle."

In consequence of this error the following changes are required:

(1) The scales for upwarp and stress in Figure 2 (p. 226) should be reduced by a factor 5.5.

(2) On p. 231, in the paragraph starting "The values of w for l = 40 km":

for "upwarp is 3·2 km" *read* "upwarp is 0·59 km"
for "120 × 10^8 dyn/cm^2" *read* "22 × 10^8 dyn/cm^2"
for "more than 30 times" *read* "more than 5 times"
for "42 × 10^8 dyn/cm^2" *read* "8 × 10^8 dyn/cm^2".

(3) On p. 232, line 11:

for "240 × 10^8 dyn/cm^2" *read* "44 × 10^8 dyn/cm^2".

(4) On p. 225, last line of Summary:

for "120 × 10^8 dyn/cm^2" *read* "22 × 10^8 dyn/cm^2".

Bernard Price Institute of
Geophysical Research,
University of the Witwatersrand,
Milner Park,
Johannesburg:
A. L. Hales
D. I. Gough

1961 *May* 17.

Book Review

Field Computations in Engineering and Physics

A. S. Thom and C. J. Apelt

(Van Nostrand, 1961, 168 pp., 30s.)

The author's object in this book is to describe, by means of theory and example, the method of "squares" for the solution of boundary-value problems in two-dimensional partial differential equations. This method has been developed and applied by Professor Thom and his colleagues (including, more recently, Dr Apelt) during the last thirty years. For example, we are indebted to Professor Thom's valuable work for much of our knowledge of steady viscous flow past bluff bodies in the range of Reynolds number below that at which a periodic wake develops.

The method of squares differs from the "relaxation" method in that the former is a straightforward iterative method, whereas in relaxation the individual worker uses his experience and skill to determine the precise step to take at a given stage of computation. For this reason the authors emphasize that the method of squares is far more suitable than the relaxation method for use with a digital computer. This is certainly an important factor in a comparison of the two methods, but it should be borne in mind that much of the work described in the book was done with a desk machine.

After two useful chapters on finite differences, several chapters of the book are devoted to the solution of Laplace's equation for various boundary conditions. Following are chapters on the solution of Poisson's equation, fourth-order linear equations, the Navier–Stokes equations of viscous flow, and the equations of compressible fluid flow. The book is liberally illustrated and has many examples, mostly taken from the fields of fluid motion and elasticity theory. It is recommended to those interested in the numerical solution of partial differential equations.

J. T. STUART.

Geophysical Journal

RECOMMENDATIONS TO AUTHORS

1. Authors are advised to follow the general suggestions for the preparation of scientific papers given in *Notes on the preparation of papers to be communicated to the Royal Society* and in *The printing of mathematics* by T. W. Chaundy, P. R. Barrett and C. Batey (Oxford University Press, 1954). In particular the abbreviations for the names of periodicals used in the *Geophysical Journal* are those given in *Notes on the preparation of papers . . .* and the symbols, signs and abbreviations are those recommended for British scientific publications by the Symbols Committee of the Royal Society.

2. Papers should be typewritten with double spacing and on one side of the paper only. Two copies, one of them the top copy, should be submitted. All papers must be accompanied by a summary; summaries are not required for *Letters to the Editors*.

3. Tables are printed without rules. They should be numbered serially with Arabic numerals. Table headings should be brief. Units should be placed at the head of the column. Tables should be typed on separate sheets and their positions in the text indicated on the copy.

4. Illustrations, especially photographs, should be kept to a minimum, and the same information should not in general be given in both tables and illustrations. Line drawings should be in dense black ink on smooth white board or transparent tracing film. Original drawings must not be lettered; lettering should be indicated on copies or photoprints which must be supplied in addition to the originals. Wherever possible, lettering should be kept outside the diagram so that it may be set in type. Diagrams should be drawn at twice to three times the size at which they will be printed. The maximum dimensions for diagrams, when reduced and with their lettering and captions, are 8 in by 5 in. A wide variety of flat tints of dots, lines and shadings can be applied to line drawings by the blockmaker. Photographs for reproduction should be unmounted glossy prints and should be accompanied by lettered prints.

5. References should be quoted in the form recommended by the Royal Society but authors may also quote titles of papers if they wish. References in the text are made in the form (Smith 1940).

6. Authors whose work involves much mathematics should follow carefully the recommendations in *The Printing of Mathematics*.

7. The Editors will be glad to advise authors on any special points of difficulty arising in the preparation of papers for the *Journal*.

Book Review

Field Computations in Engineering and Physics

A. S. Thom and C. J. Apelt

(Van Nostrand, 1961, 168 pp., 30s.)

The author's object in this book is to describe, by means of theory and example, the method of "squares" for the solution of boundary-value problems in two-dimensional partial differential equations. This method has been developed and applied by Professor Thom and his colleagues (including, more recently, Dr Apelt) during the last thirty years. For example, we are indebted to Professor Thom's valuable work for much of our knowledge of steady viscous flow past bluff bodies in the range of Reynolds number below that at which a periodic wake develops.

The method of squares differs from the "relaxation" method in that the former is a straightforward iterative method, whereas in relaxation the individual worker uses his experience and skill to determine the precise step to take at a given stage of computation. For this reason the authors emphasize that the method of squares is far more suitable than the relaxation method for use with a digital computer. This is certainly an important factor in a comparison of the two methods, but it should be borne in mind that much of the work described in the book was done with a desk machine.

After two useful chapters on finite differences, several chapters of the book are devoted to the solution of Laplace's equation for various boundary conditions. Following are chapters on the solution of Poisson's equation, fourth-order linear equations, the Navier–Stokes equations of viscous flow, and the equations of compressible fluid flow. The book is liberally illustrated and has many examples, mostly taken from the fields of fluid motion and elasticity theory. It is recommended to those interested in the numerical solution of partial differential equations.

J. T. Stuart.

Geophysical Journal

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1. Authors are advised to follow the general suggestions for the preparation of scientific papers given in *Notes on the preparation of papers to be communicated to the Royal Society* and in *The printing of mathematics* by T. W. Chaundy, P. R. Barrett and C. Batey (Oxford University Press, 1954). In particular the abbreviations for the names of periodicals used in the *Geophysical Journal* are those given in *Notes on the preparation of papers . . .* and the symbols, signs and abbreviations are those recommended for British scientific publications by the Symbols Committee of the Royal Society.

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